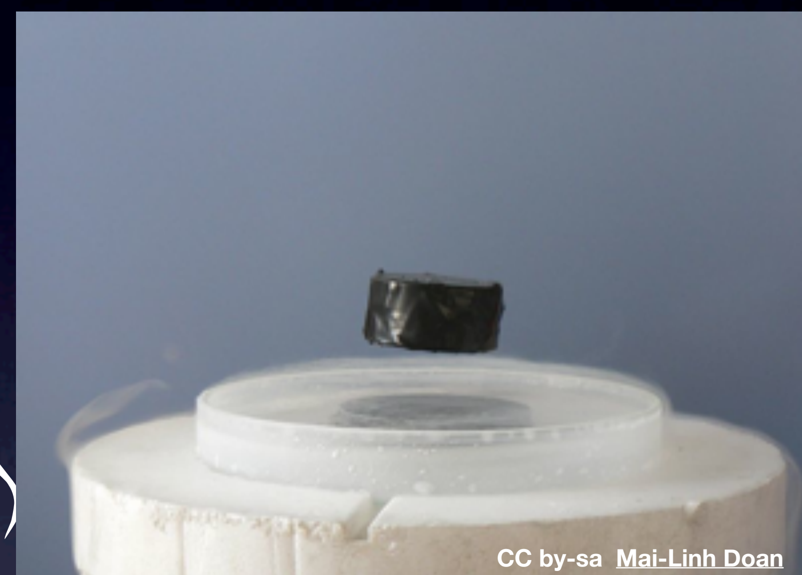
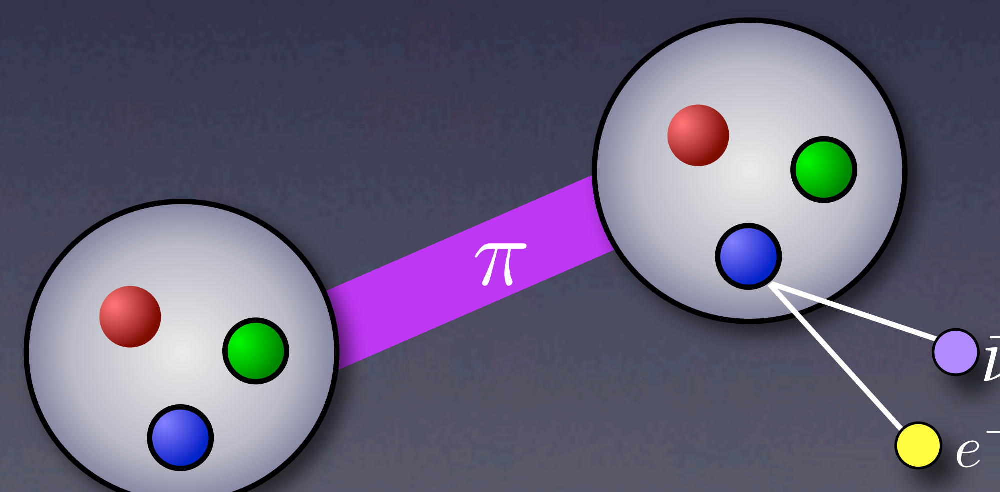
Generalization of Nambu-Goldstone theorem to nonrelativistic systems

Yoshimasa Hidaka

(Nishina Center, RIKEN)

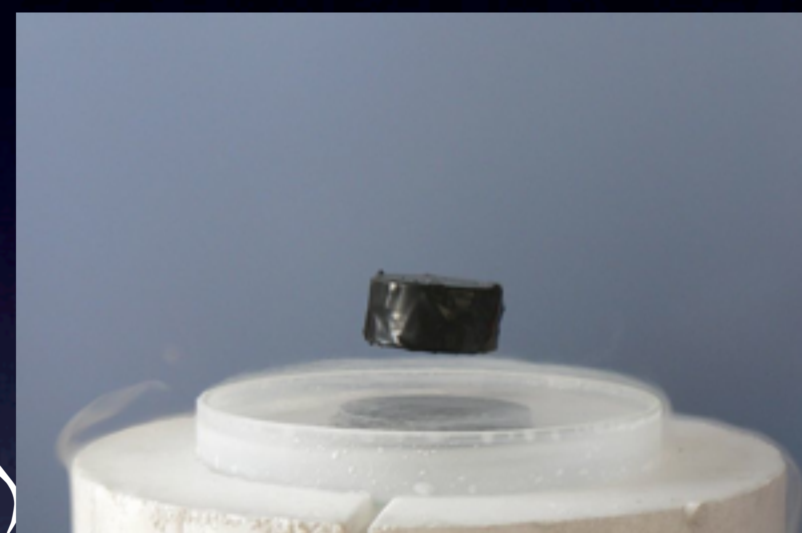
Several physical phenomena

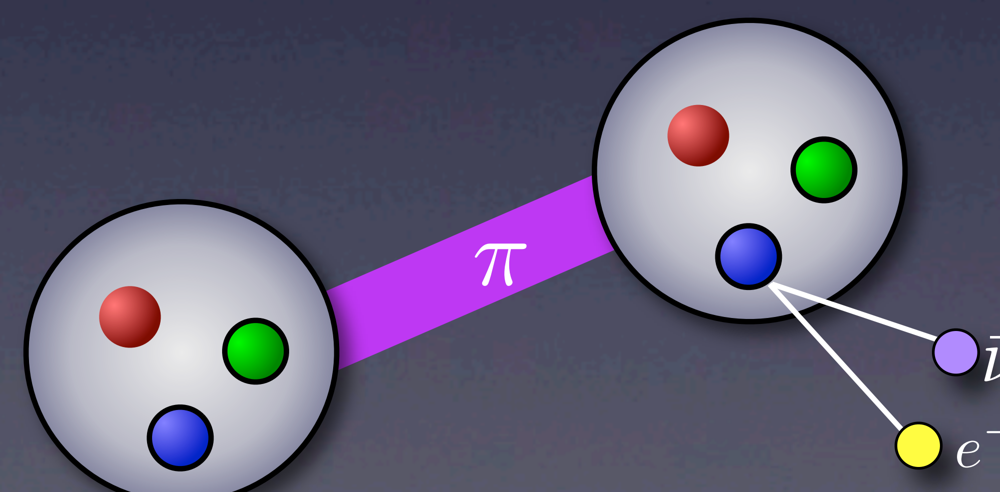




Several physical phenomena

Spontaneous symmetry breaking





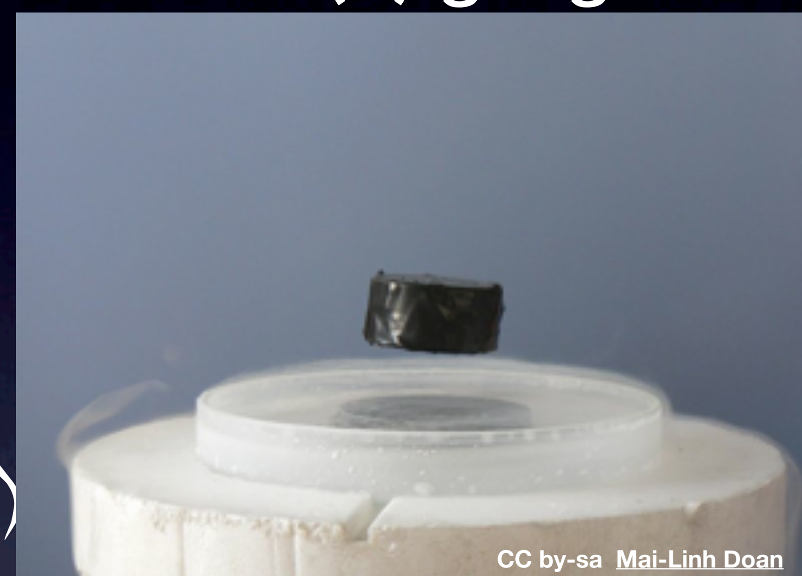
Several physical phenomena

Spontaneous symmetry breaking

Translation

CC by-sa Didier Descouens

U(1) gauge



CC by-sa Mai-Linh Doan

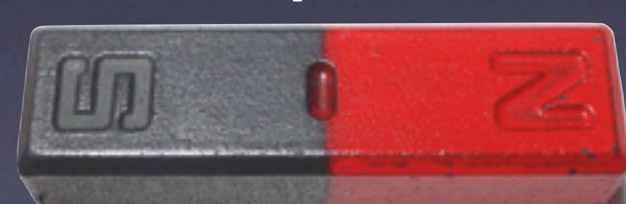
translation

CC by-sa Roger McLassus

Galilei

CC by-sa Elijah van der Giessen

spin



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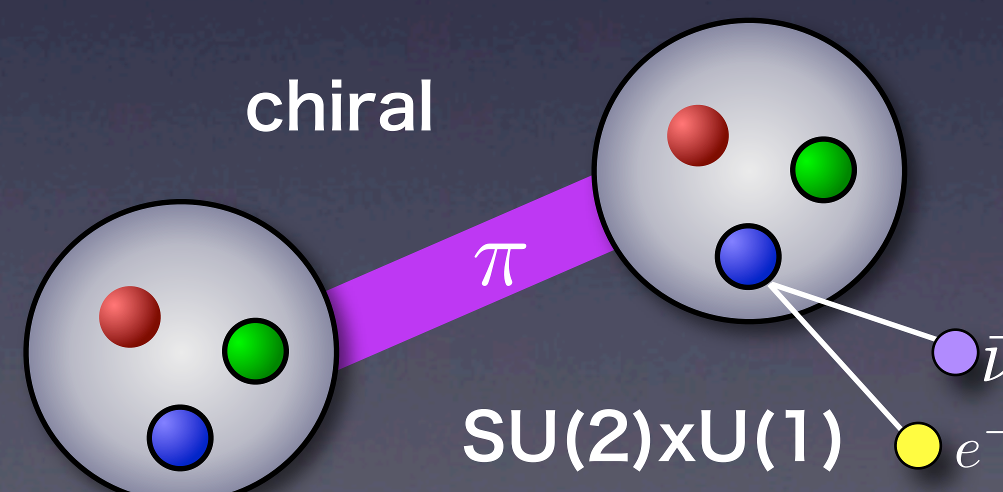
chiral



SU(2)xU(1)
gauge

Several physical phenomena

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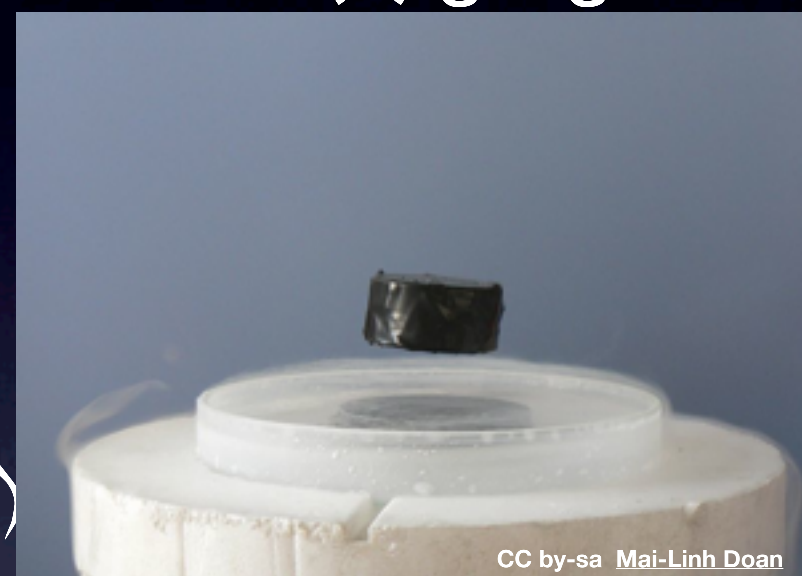
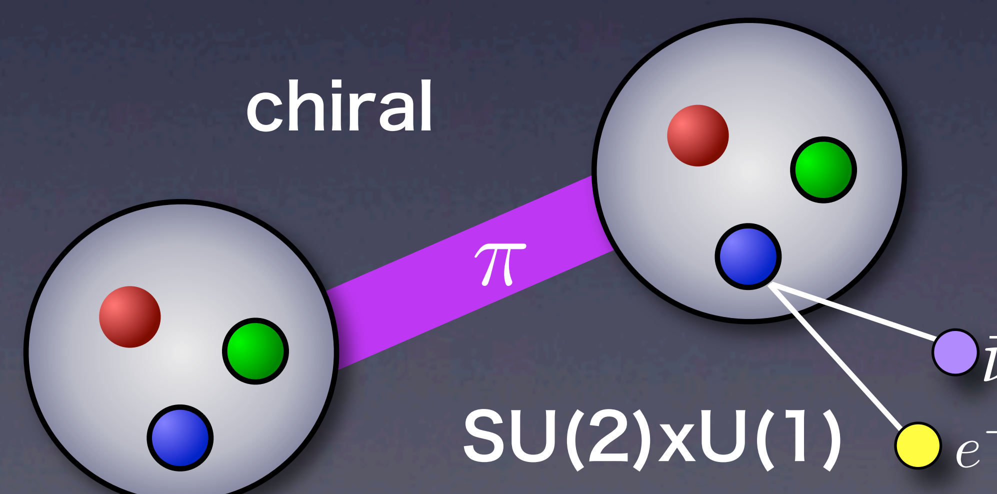


translation

Galilei

spin

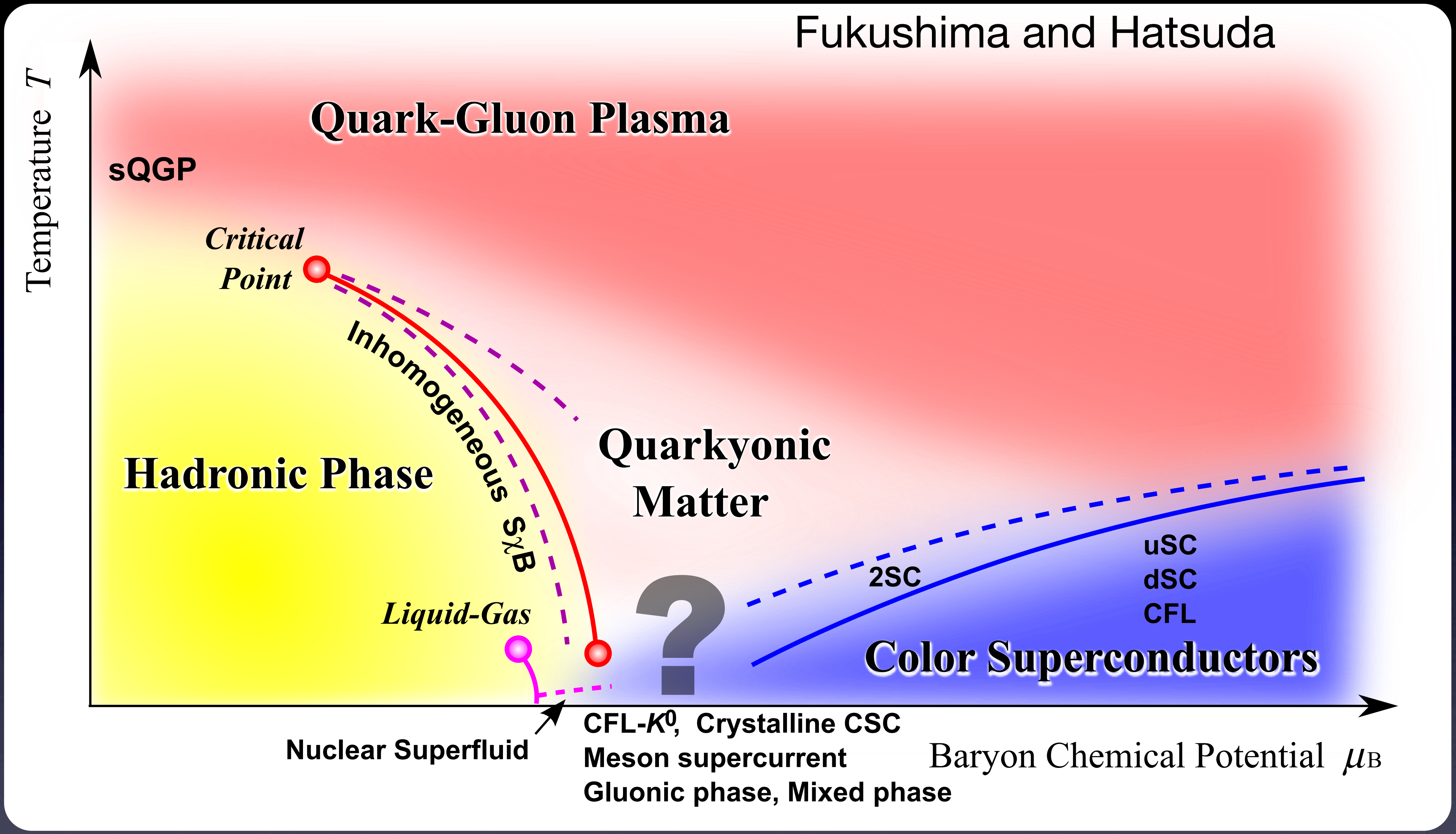
chiral



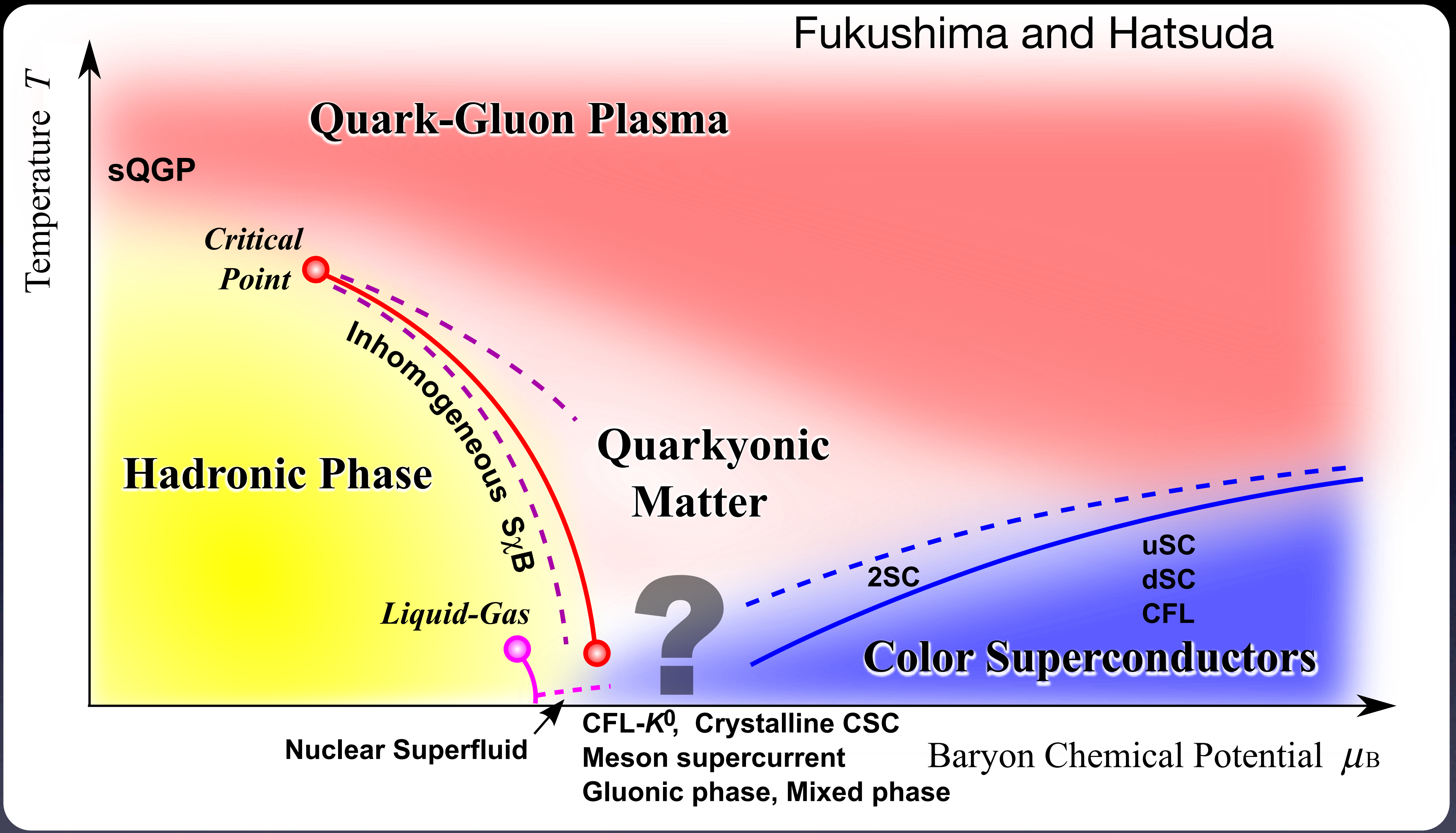
SU(2)xU(1) gauge

In many cases, waves appear.

QCD phase diagram



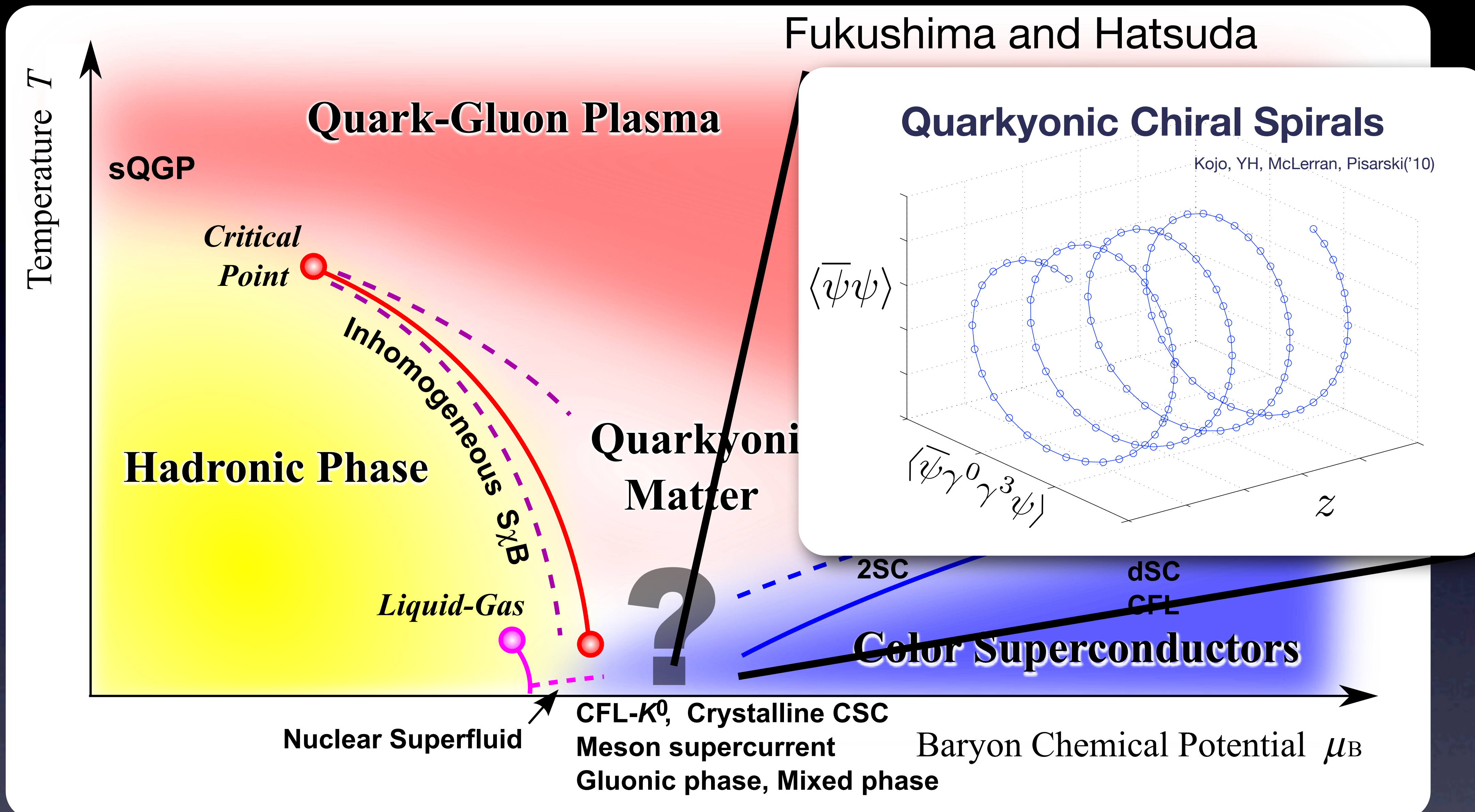
QCD phase diagram



chiral symmetry breaking, translation breaking, rotation breaking.....

What is the low-energy excitation?

QCD phase diagram



chiral symmetry breaking, translation breaking, rotation breaking.....

What is the low-energy excitation?

NG modes in QCD

Pion

SSB of chiral symmetry

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

$$N_{\mathrm{BS}} = 3, \quad N_{\mathrm{NG}} = 3$$

Dispersion: $\omega = k$

NG modes in QCD

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Dispersion: $\omega = k$

NG modes in Kaon condensed CFL phase

Miransky, Shovkovy ('02) Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$SU(2)_I \times U(1)_Y \to U(1)_{\rm em}$$

$$N_{\rm BS} = 3, \quad N_{\rm NG} = 2$$

Dispersion: $\omega = k^2$

Plan of my talk

What are low-energy excitations

Plan of my talk

What are low-energy excitations

for internal symmetries breaking?

Plan of my talk

What are low-energy excitations

for internal symmetries breaking?

for spacetime symmetries breaking?

Symmetry and conservation law

Noether's theorem   Noether 1915

Symmetry		Conserved charges
Time translation	⟺	Energy
Spatial translation		Momentum
Rotation		Angular momentum
U(1) phase		Charge

Symmetry and conservation law

Noether's theorem   Noether 1915

Symmetry		**Conserved charges**
Time translation	$\Leftrightarrow$	Energy
Spatial translation		Momentum
Rotation		Angular momentum
U(1) phase		Charge

$\Rightarrow$

$$\partial_t n_a(t,\boldsymbol{x}) + \partial_i j^i_a(t,\boldsymbol{x}) = 0$$

Charge $Q_a = \int d^3x n_a(t,\boldsymbol{x})$ $\dfrac{d}{dt}Q_a = 0$

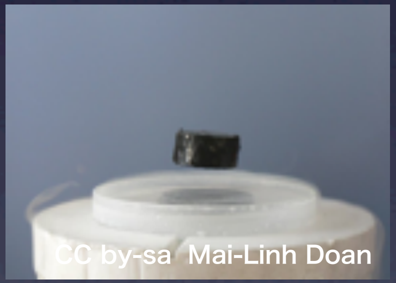
Pattern of symmetry breaking

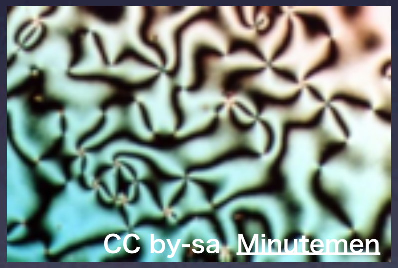
Explicit

Parity breaking, CP breaking ...

Spontaneous

Magnet Superconductor crystal liquid crystal, ...





Anomaly

Chiral, Weyl, gauge, parity anomalies,....

Continuum symmetry

Internal

Isospin

proton neutron

Spin of electron

up down

Spacetime

Space-time translations, rotation, boost

Gauge

Electroweak and strong

U(1)xSU(2)xSU(3)

Spontaneous symmetry breaking

Spontaneous breaking:
there exists at least one local operator, Φ_i, such that

$$\langle [Q_a, \phi_i(\boldsymbol{x})] \rangle \equiv \mathrm{tr}\rho\, [Q_a, \phi_i(\boldsymbol{x})] \neq 0$$

Vacuum: $\rho = |\Omega\rangle\langle\Omega|$

In medium: $\rho = \dfrac{\exp(-\beta(H - \mu N))}{\mathrm{tr}\exp(-\beta(H - \mu N))}$

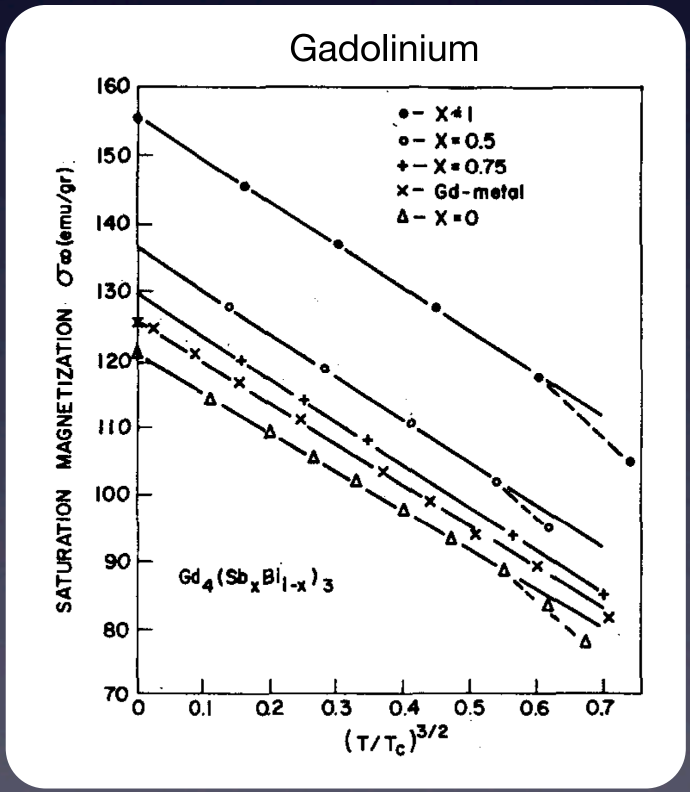
Spontaneous symmetry breaking

Why important?

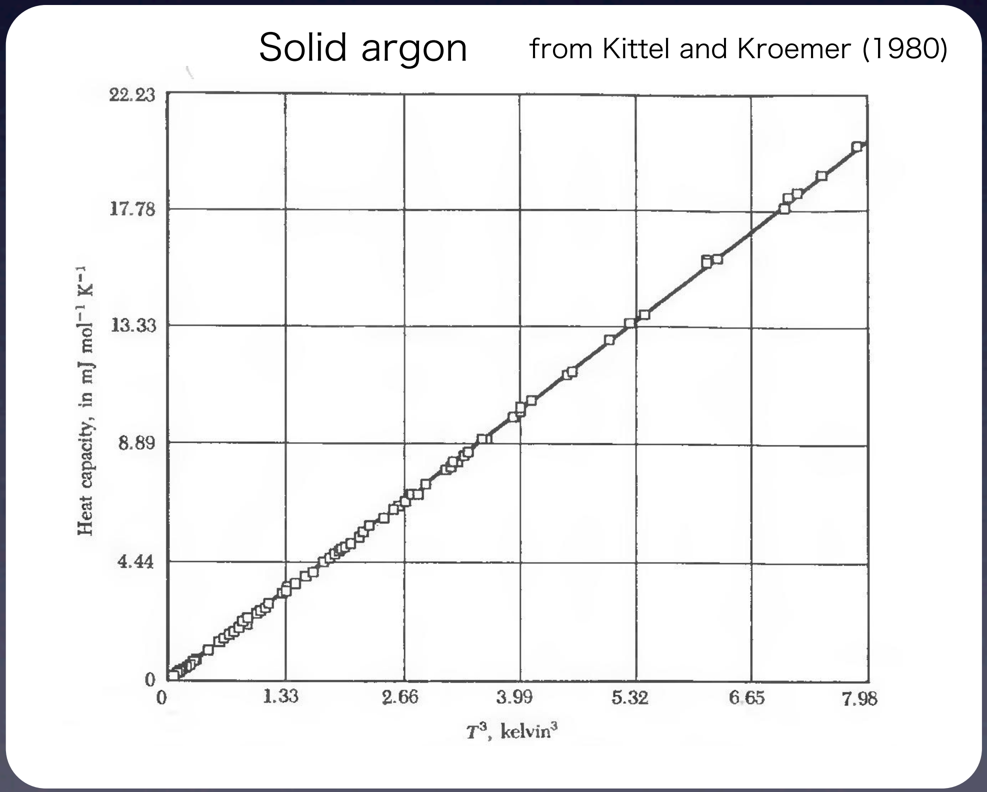
Without detail of systems, one can predict many things:

dispersion relations, low-energy theorem,...

Bloch $T^{3/2}$ law, Debye T^3 law, ...



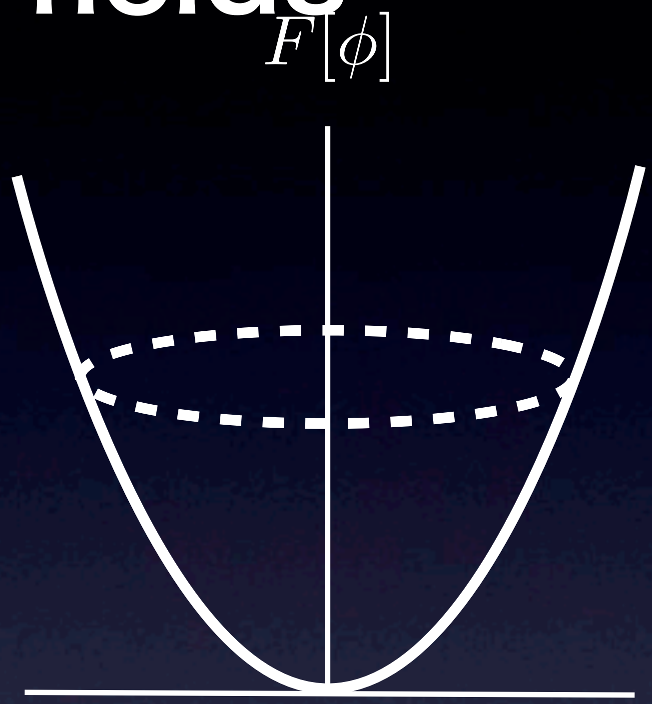
Holtzberg, McGuire, M'ethfessel, Suits, J. Appl. Phys. 35,1033 (1964)



Chiral condensate $$\frac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_0} = 1 - \frac{1}{8}\frac{T^2}{f_\pi^2} + \cdots$$

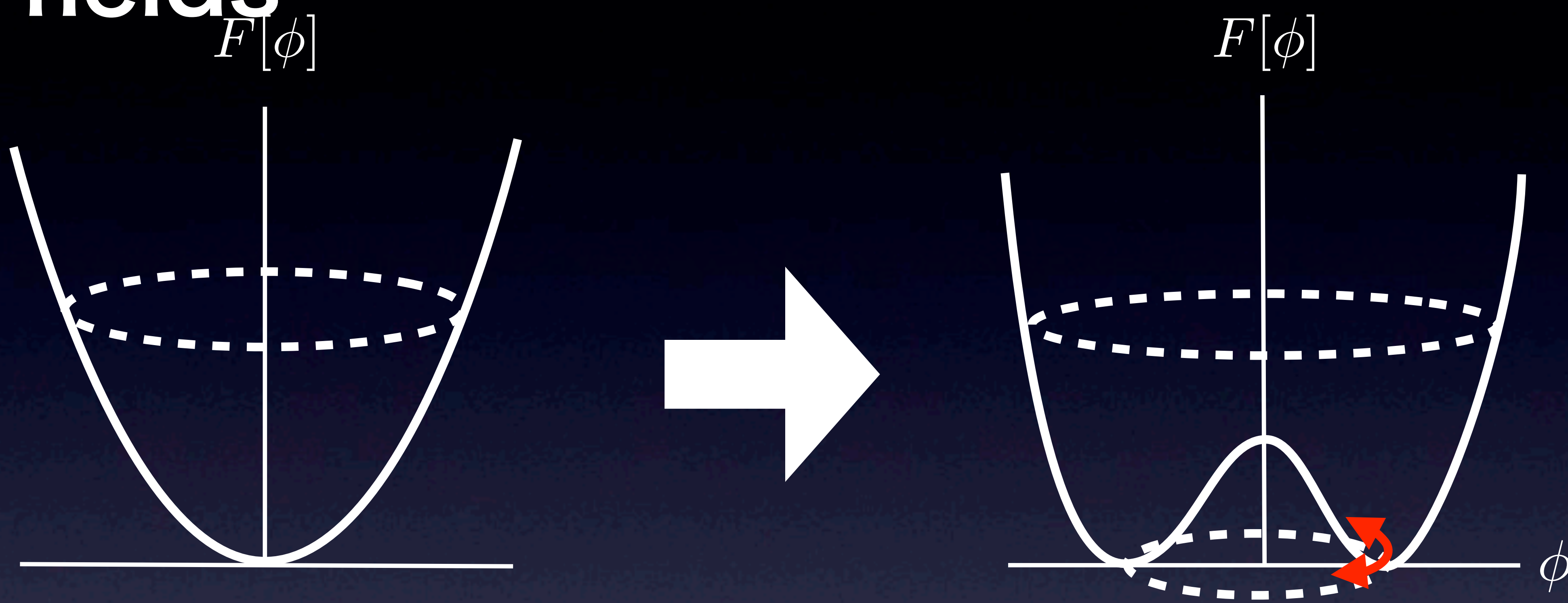
Spontaneous symmetry breaking

For fields



Spontaneous symmetry breaking

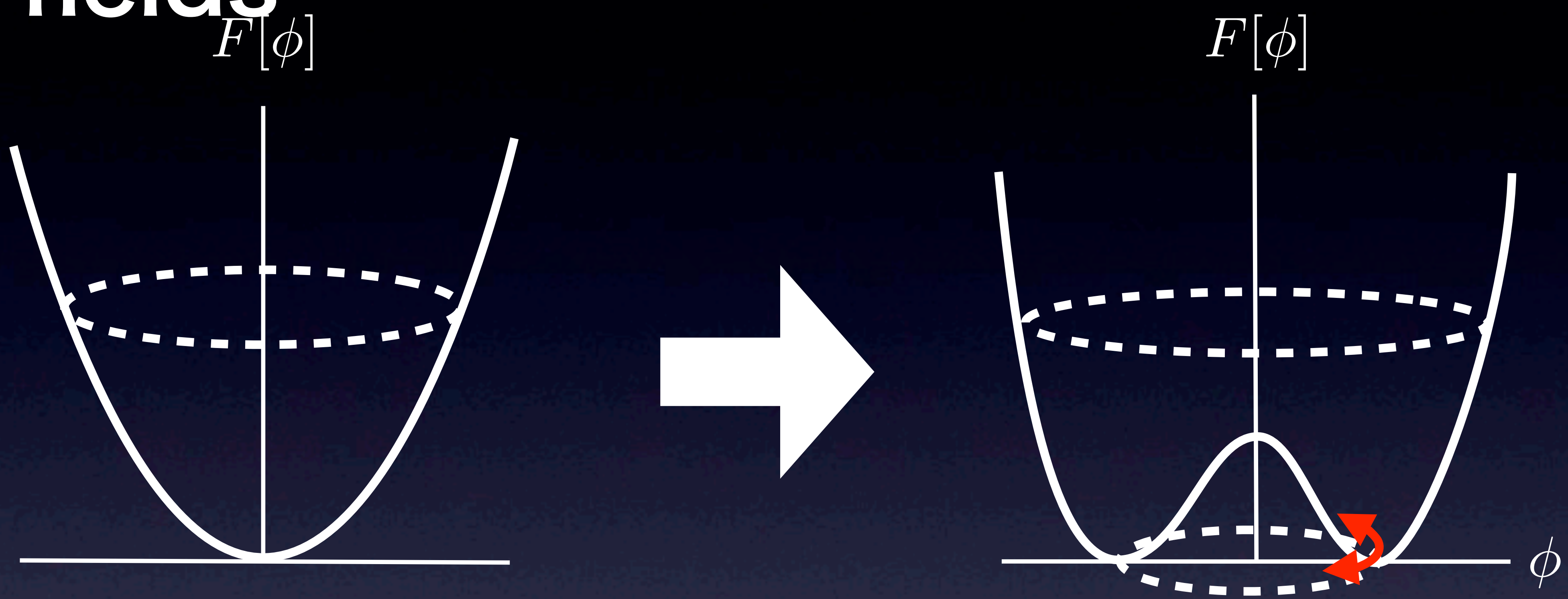
For fields



Degeneracy of grand states

Spontaneous symmetry breaking

For fields

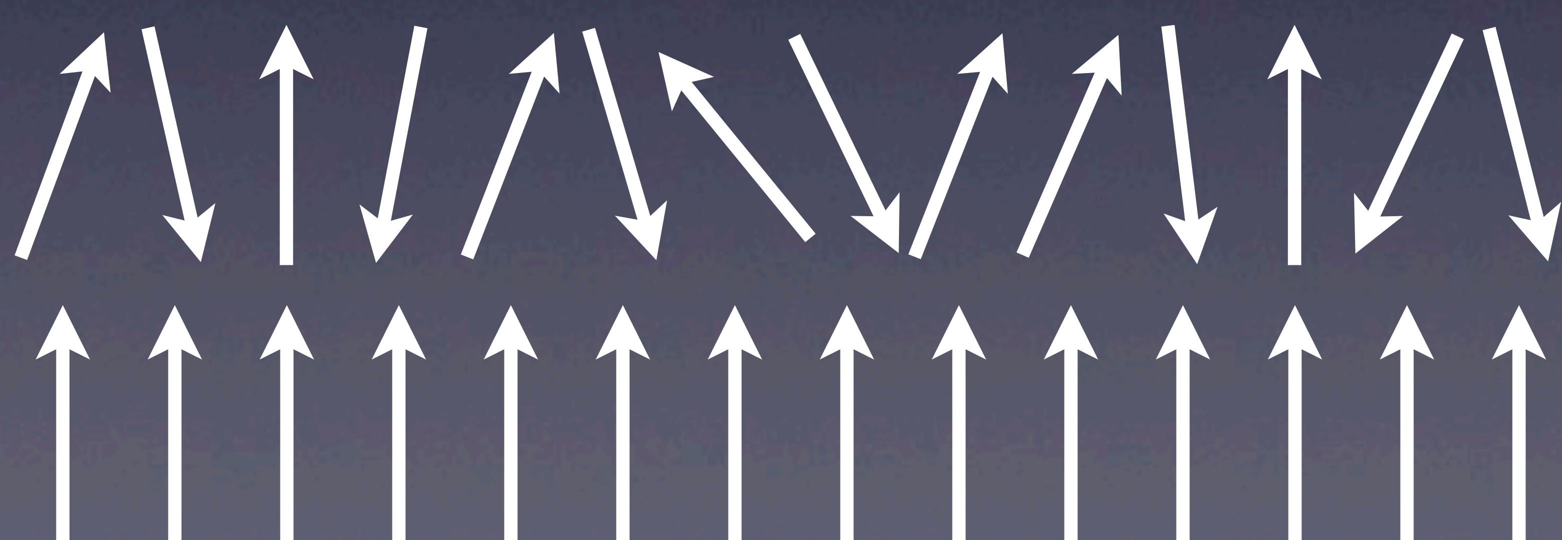


Degeneracy of grand states

For spins

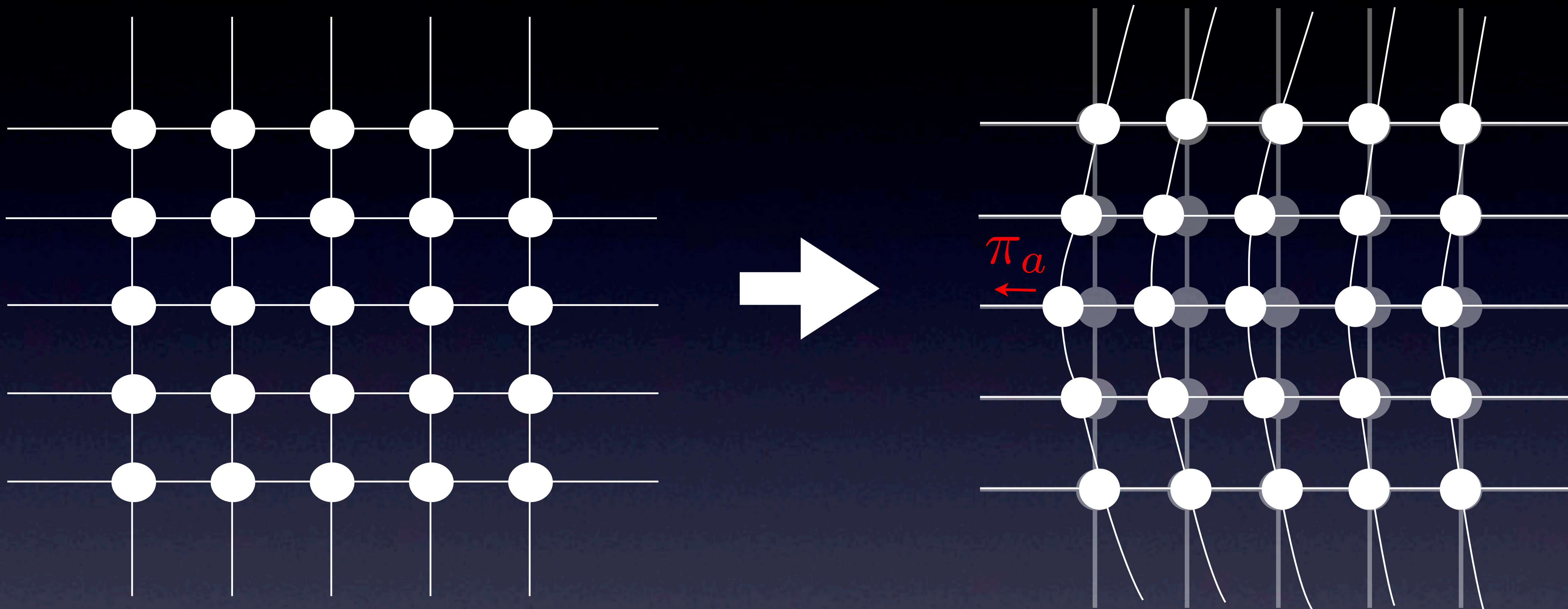
random

aligned

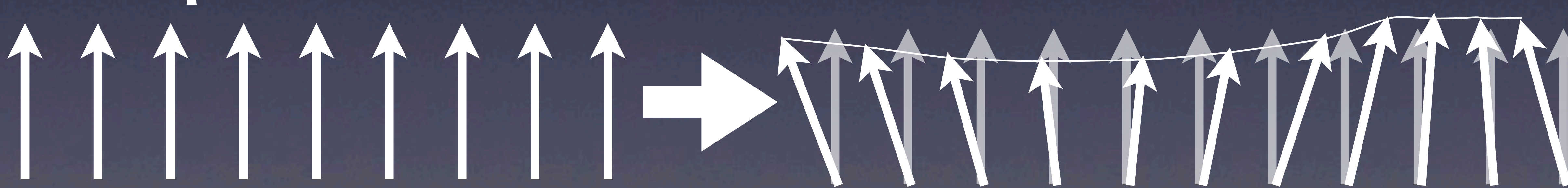


Elasticity

For crystal



For spin

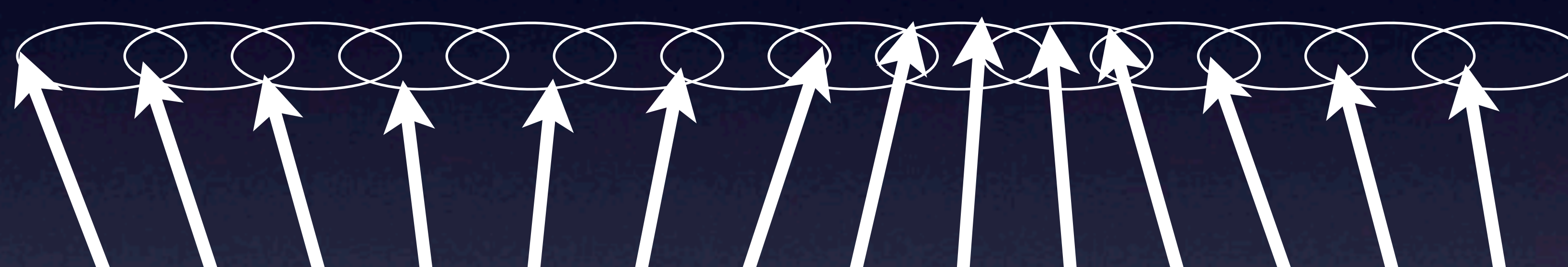


Free energy $F = \frac{1}{2}(\partial_i \pi^a)^2 + \cdots$

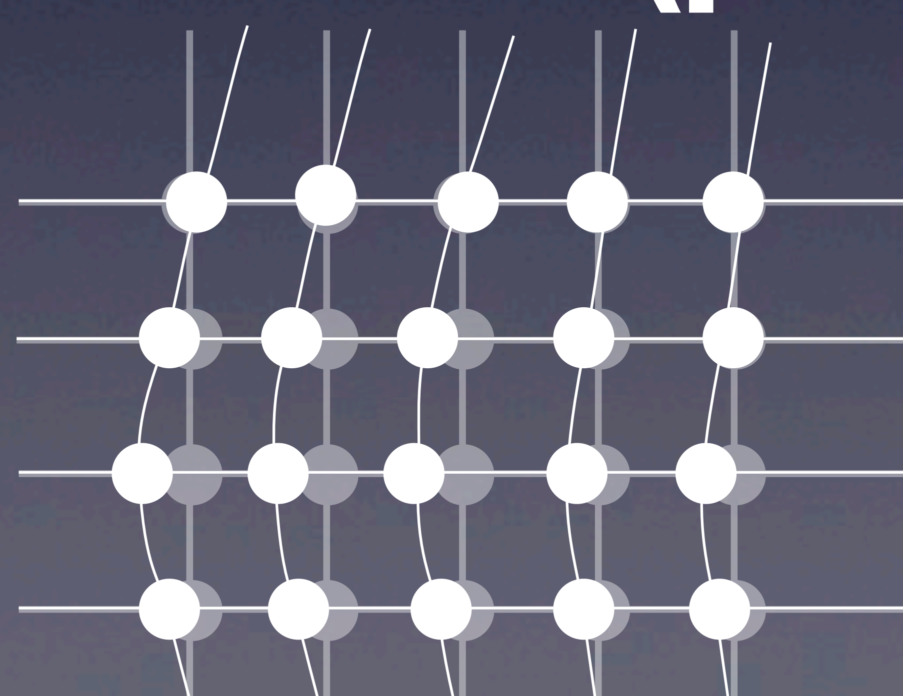
Gapless excitations
= Nambu-Goldstone (NG) mode

Nambu('60), Goldstone(61), Nambu, Jona-Lasinio('61),

Spin wave (magnon)



Crystal vibration (phonon)



Nambu-Goldstone theorem

Goldstone, Salam, Weinberg('62)

For Lorentz invariant vacuum

Spontaneous breaking of global symmetry

⬇

# of broken symmetry = # of NG modes

Dispersion relation $\omega = c|\boldsymbol{k}|$

Example in relativistic systems

Approximate symmetry of QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

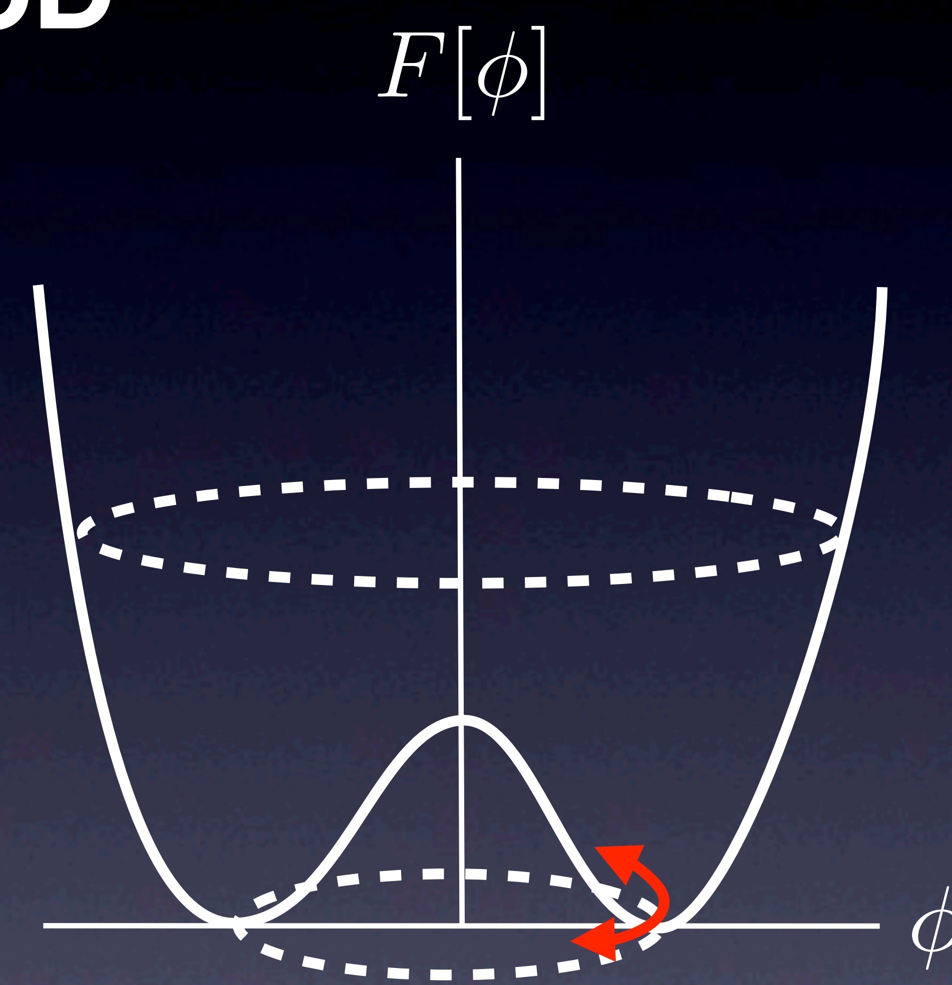
Three broken generators

Three NG modes: Pions

$$\pi^+, \pi^-, \pi^0$$

Dispersion relation

$$\omega = \sqrt{\boldsymbol{k}^2 + m_\pi^2}$$



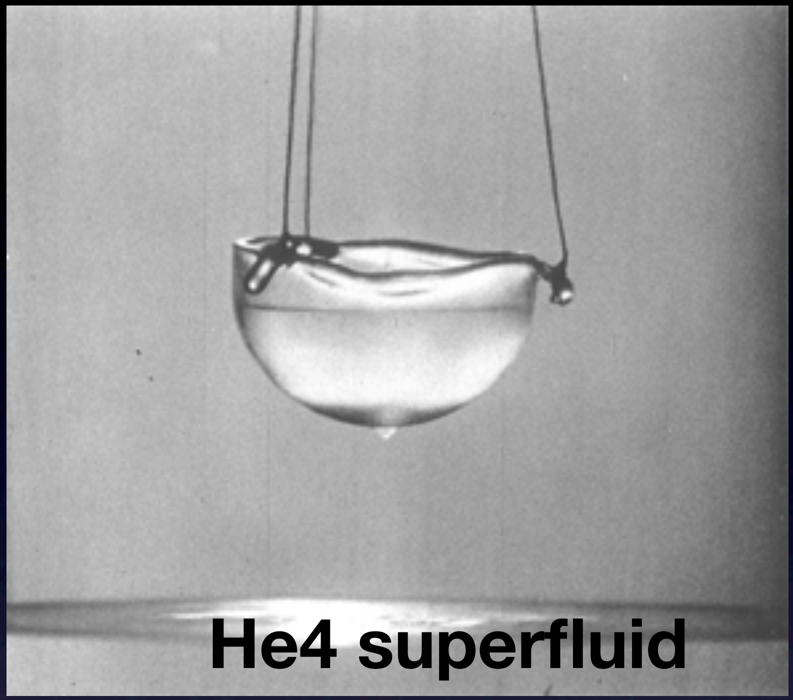
Example of NG modes

Superfluid phonon

broken of number

Broken generator: Q

One phonon $\omega \sim |\boldsymbol{k}|$

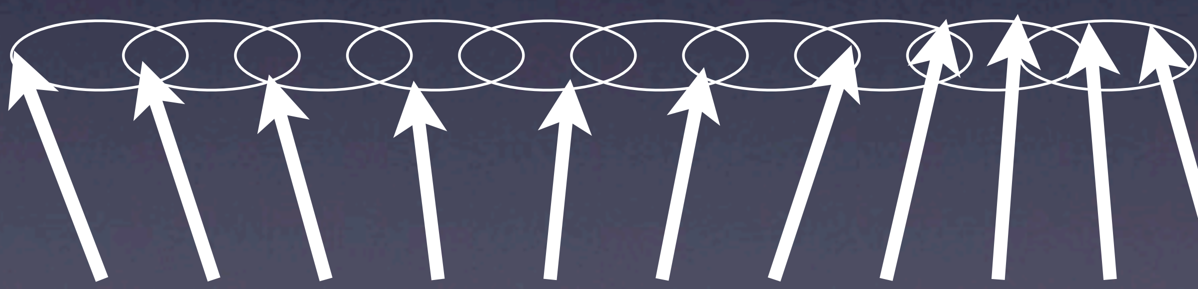


Magnon

Broken of rotation

Two broken generators S_x, S_y

one magnon $\omega \sim \boldsymbol{k}^2$



 # and dispersion are different from relativistic ones

Generalization

Nielsen - Chadha ('76)

$$N_{\text{type-I}} + 2N_{\text{type-II}} \geq N_{\text{BS}}$$

Type-I: $\omega \propto k^{2n+1}$ **Type-II:** $\omega \propto k^{2n}$

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Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$\langle [Q_a, Q_b] \rangle = 0 \Rightarrow N_{\text{NG}} = N_{\text{BS}}$$

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$$\langle [Q_a, Q_b] \rangle = 0 \Rightarrow N_{\text{NG}} = N_{\text{BS}}$$

Watanabe - Brauner ('11)

$$N_{\text{BS}} - N_{\text{NG}} \leq \frac{1}{2} \text{rank} \langle [Q_a, Q_b] \rangle$$

Recent progress

Effective Lagrangian method **Watanabe, Murayama ('12)**

Mori's projection operator method **YH ('12)**

- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[Q_a, Q_b]\rangle$
- $N_{\text{type-I}} + 2N_{\text{type-II}} = N_{\mathrm{BS}}$
- $N_{\text{type-II}} = \frac{1}{2}\mathrm{rank}\langle[Q_a, Q_b]\rangle$

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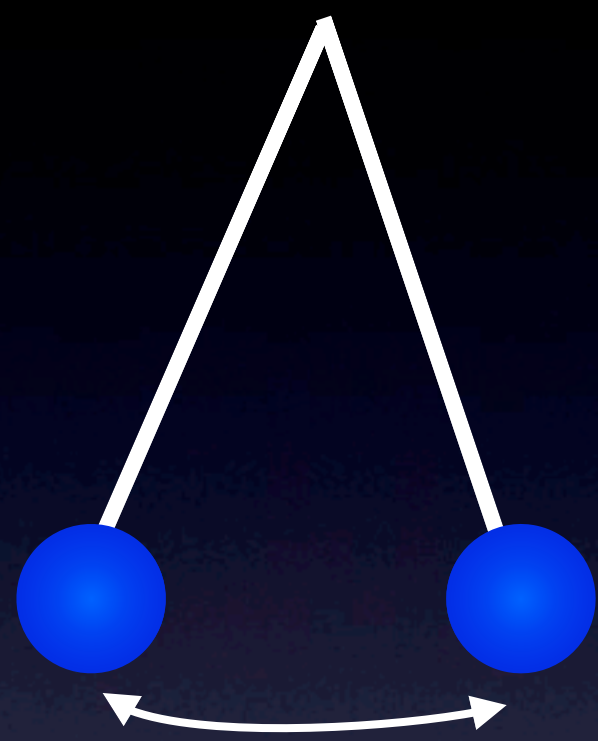
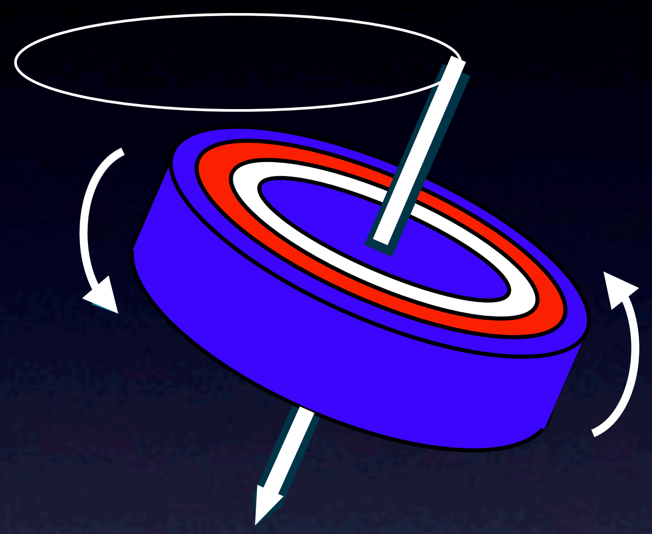
- $N_{\text{type-B}} = \frac{1}{2}\mathrm{rank}\langle[Q_a, Q_b]\rangle$

Two type of excitations





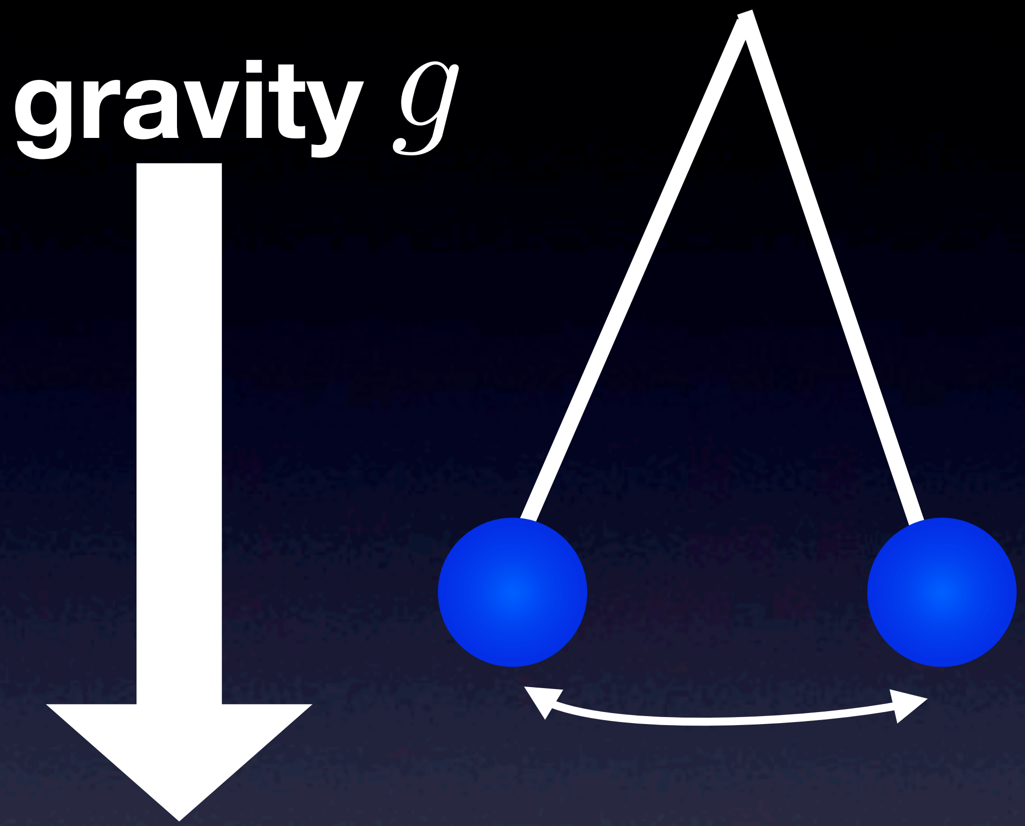
Type-A

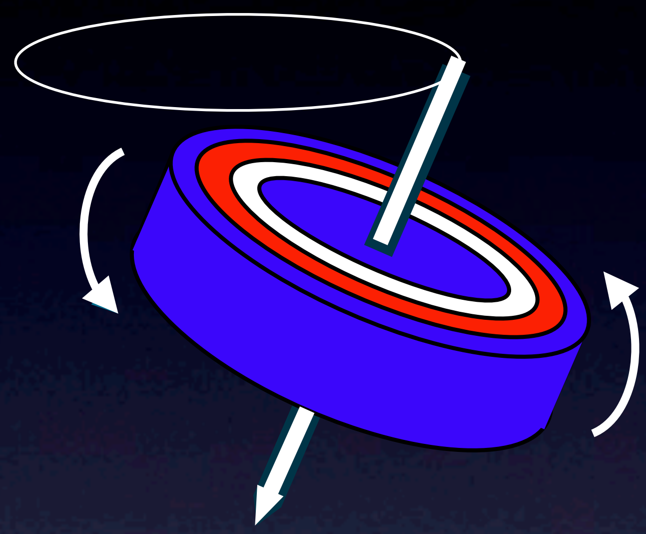
Harmonic oscillation

Type-B

Precession

Two type of excitations





Type-A

Harmonic oscillation

$$\omega \sim \sqrt{g}$$

Type-B

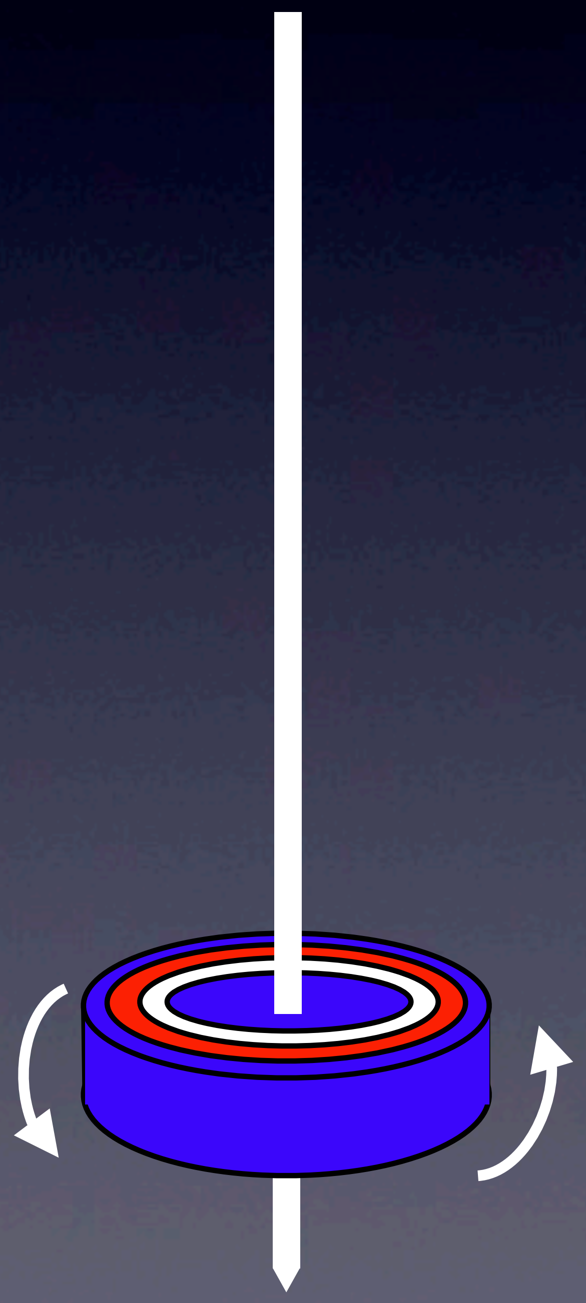
Precession

$$\omega \sim g$$

Intuitive example for type-II NG modes

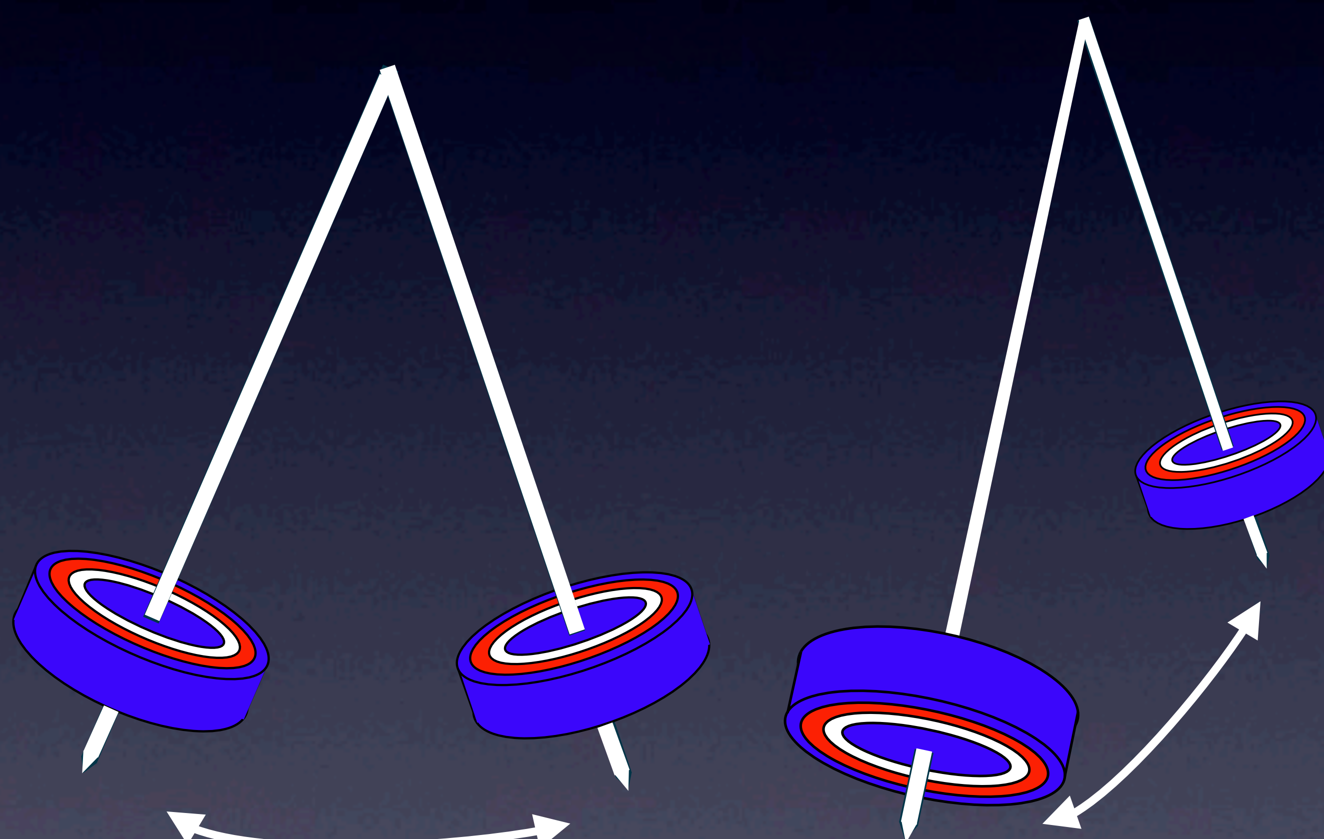
Pendulum with a spinning top

- Rotation symmetry is explicitly broken by a weak gravity
- Rotation along with z axis is unbroken.
- Rotation along with x or y is broken.
- The number of broken symmetry is two.



Intuitive example for type-II NG modes

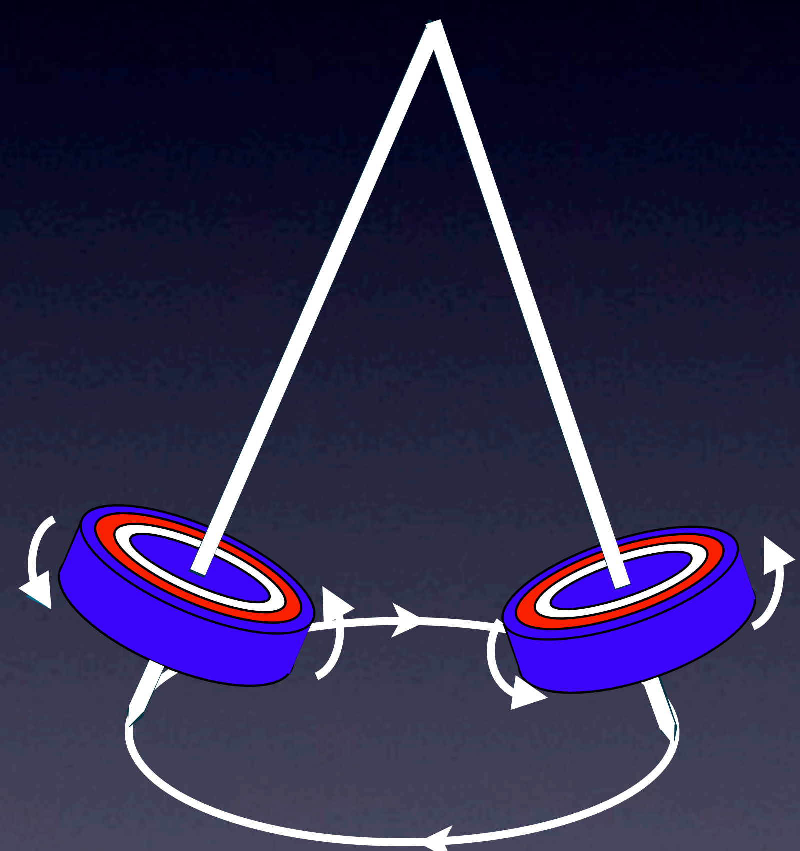
Pendulum has two oscillation motions



if the top is not spinning.

Intuitive example for type-II NG modes

If the top is spinning,



the only one rotation motion (Precession) exists.

In this case, $\{L_x, L_y\}_P = L_z \neq 0$

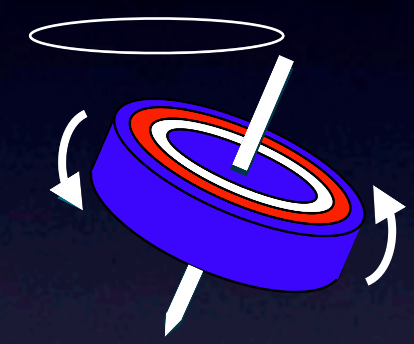
Recent Progress

Watanabe, Murayama ('12), YH ('12)

NG modes associated with spontaneous breaking of internal symmetry can be classified by two types:

Type-A

harmonic oscillation



Type-B

precession

$$N_{\text{type-A}} = N_{\text{BS}} - 2N_{\text{type-B}} \qquad N_{\text{type-B}} = \frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$$

- $N_{\text{BS}} - N_{\text{NG}} = \frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$

What is the NG mode?

charge densities are slow:

$$\partial_t n_a(t, \boldsymbol{x}) = -\partial_i j^i_a(t, \boldsymbol{x})$$

What is the NG mode?

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$$\partial_t n_a(t,\boldsymbol{x}) = -\partial_i j_a^i(t,\boldsymbol{x})$$

ex) in medium $j_a^i = \Gamma\partial_i n_a$

Diffusion equation $\partial_t n_a(t,\boldsymbol{x}) = -\Gamma\partial_i^2 n_a(t,\boldsymbol{x})$

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Diffusion equation $\partial_t n_a(t,\boldsymbol{x}) = -\Gamma \partial_i^2 n_a(t,\boldsymbol{x})$

When SSB occurs,

the charge density and the local operator are canonically conjugate cf. Nambu ('04)

$$\langle [iQ_a, \pi_b(\boldsymbol{x})] \rangle \neq 0$$

$$\Rightarrow \quad \begin{aligned} \partial_t \pi_a &= c n_a \\ \partial_t n_a &= b \partial_i^2 \pi_a \end{aligned}$$

Is Type-A (B) mode Type-I (II) NG mode？

Type-A NG modes

Pair: charge density and local operator

$$\langle [iQ_a, \pi_b(\boldsymbol{x})] \rangle \neq 0$$

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Type-A NG modes

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$$\partial_t \pi_a = c n_a \quad \partial_t n_a = d \partial_i^2 \pi_a$$

$$\Rightarrow \quad \omega = \sqrt{cd} |\boldsymbol{k}|$$

Type-A = Type-I

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Type-A NG modes

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Type-A = Type-I

Type-B NG modes

Pair: charge densities

$$\langle [iQ_a, n_b(\boldsymbol{x})] \rangle \neq 0$$

$$\partial_t n_a = c' \partial_i^2 n_b \quad \partial_t n_b = -d' \partial_i^2 n_a$$

$$\Rightarrow \omega = \sqrt{c' d'} \boldsymbol{k}^2$$

Type-B = Type-II

Is Type-A (B) mode Type-I (II) NG mode？

Type-A NG modes

Pair: charge density and local operator

$$\langle [iQ_a, \pi_b(\boldsymbol{x})] \rangle \neq 0$$

$$\partial_t \pi_a = c n_a \quad \partial_t n_a = d \partial_i^2 \pi_a$$

$$\Rightarrow \omega = \sqrt{cd}|\boldsymbol{k}| + \Gamma \boldsymbol{k}^2$$ **Type-A = Type-I**

Type-B NG modes

Pair: charge densities

$$\langle [iQ_a, n_b(\boldsymbol{x})] \rangle \neq 0$$

$$\partial_t n_a = c' \partial_i^2 n_b \quad \partial_t n_b = -d' \partial_i^2 n_a$$

$$\Rightarrow \omega = \sqrt{c' d'} \boldsymbol{k}^2 + \Gamma |\boldsymbol{k}|^4$$ **Type-B = Type-II**

Effective Lagrangian approach

Leutwyler('94) Watanabe, Murayama ('12)

Write down all possible term

$$\mathcal{L} = \frac{1}{2}\rho_{ab}\pi^a\dot{\pi}^b + \frac{\bar{g}_{ab}}{2}\dot{\pi}^a\dot{\pi}^b - \frac{g_{ab}}{2}\partial_i\pi^a\partial_i\pi^b$$

+higher orders

Effective Lagrangian approach

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+higher orders

No Lorentz symmetry:

The first derivative term may appear.

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+higher orders

No Lorentz symmetry:

The first derivative term may appear.

Lagrangian is invariant under symmetry transformation up to surface term.

$$\rho_{ab} \propto -i\langle[Q_a, j_b^0(x)]\rangle$$

Watanabe, Murayama ('12)

Examples of Type-B NG modes

	N_{BS}	$N_{\text{type-A}}$	$N_{\text{type-B}}$	$\frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$	$N_{\text{type-A}} + 2N_{\text{type-B}}$
Spin wave in ferromanget O(3)→O(2)	2	0	1	1	2
NG modes in Kaon condensed CFL $SU(2)xSU(1)_Y \to U(1)_{em}$	3	1	1	1	3
Kelvin waves in vortex translation $\mathbf{R}^3 \to \mathbf{R}^1$	2	0	1	1	2
nonrelativistic massive CP^1 model $U(1)x\mathbf{R}^3 \to \mathbf{R}^2$	2	0	1	1	2

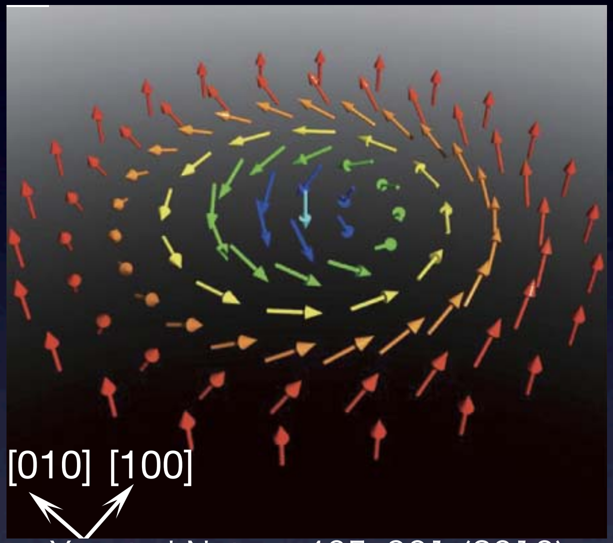
$$N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\text{BS}} \qquad N_{\text{BS}} - N_{\text{NG}} = \frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$$

Topological soliton and central extension

Translations

Ex) 2+1D skyrmion

Watanabe, Murayama 1401.8139



Yu, et al Nature 465, 901 (2010)

$$[P_x, P_y] \propto N$$

x translation   y translation   topological number

Translation and internal symm.

例) domain wall in nonrelativistic massive CP^1 model

Kobayashi, Nitta 1402.6826

$$[P_z, Q] \propto N$$

z-translation U(1) charge   topological number

SSB wit a small breaking term

$$H = H_0 + hV$$

Symmetric small explicit breaking term

Pseudo NG modes

Type-A: $\omega \sim \sqrt{h}$

Ex) pions

Type-B: $\omega \sim h$

Ex) magnon in an external magnetic field

No higher corrections if the explicit breaking term is a charge.

Nicolis, Piazza ('12), ('13)
Watanabe, Brauner, Murayama ('13)

Spontaneous breaking of spacetime symmetry

Two type of conserved charges

Translationally invariant

$$[P_\mu, Q_a] = 0$$

translational operator charge

Ex: Translationally invariant charges

Spacetime translation, chiral symmetry,

flavor symmetry, ...

Ex: Non-translationally invariant charges

Rotation, boost, conformal,

residual gauge symmetry in the covariant gauge,...

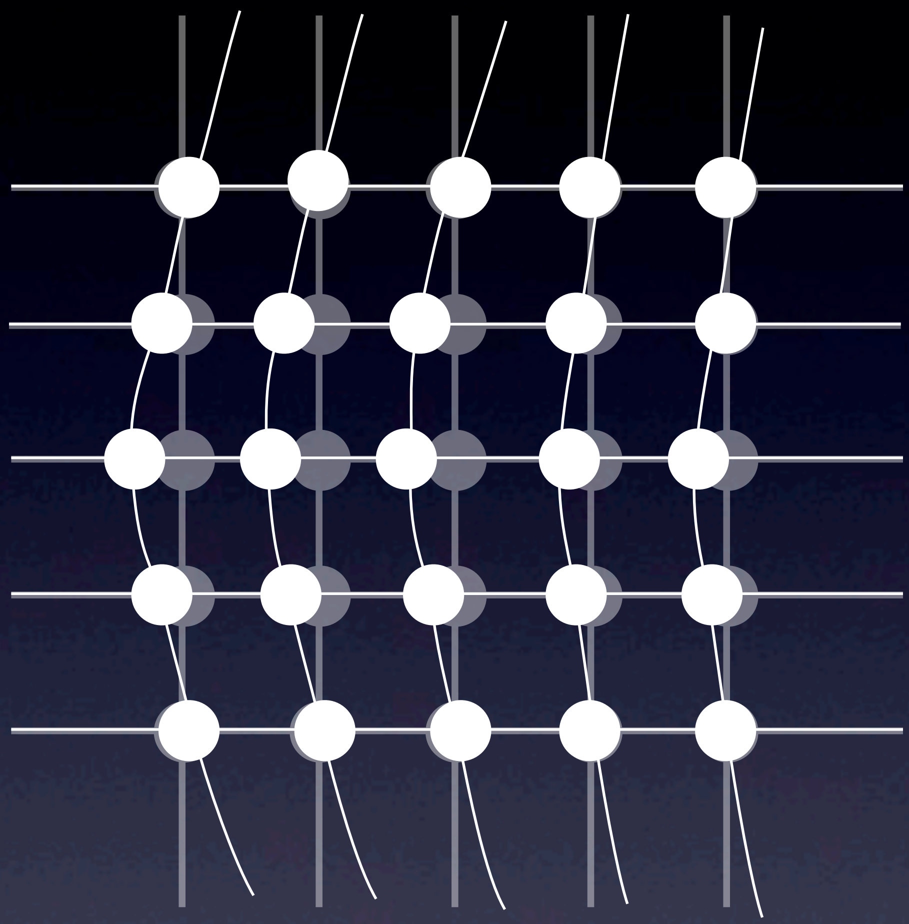
Example of spacetime breaking

Crystal vibration

Translation(3), Rotation(3), Boost(3)

9 breaking,

but three NG modes appear.



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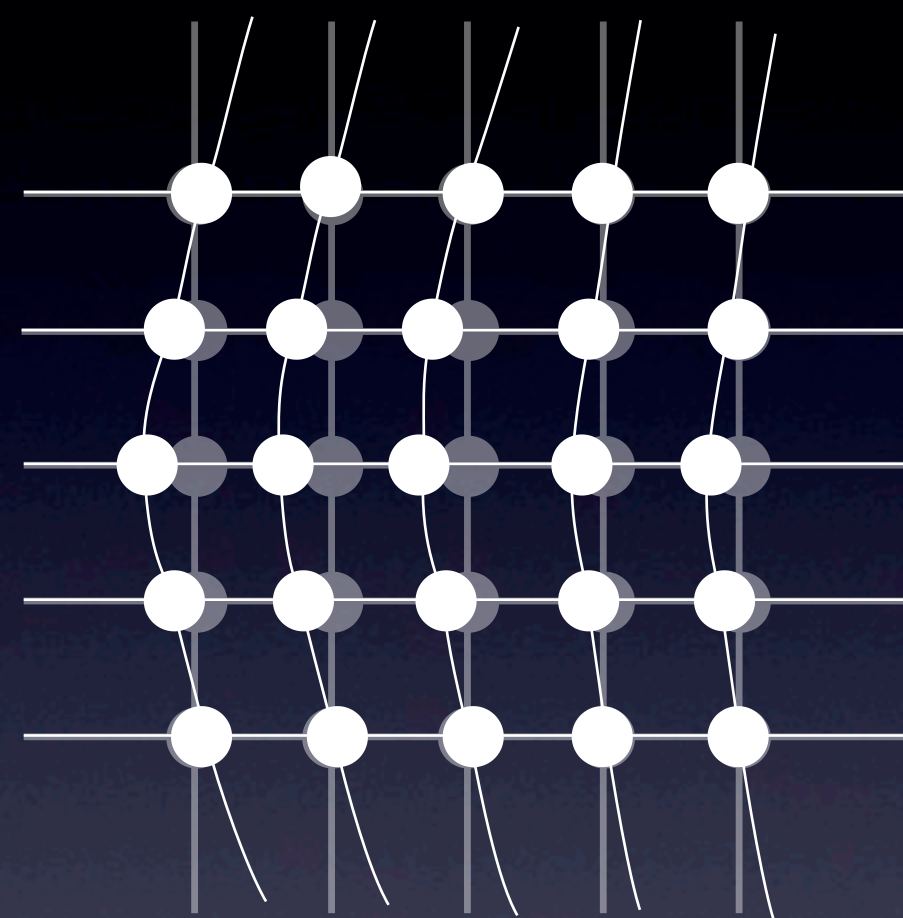

Where are NG mode associated with rotations and boosts?

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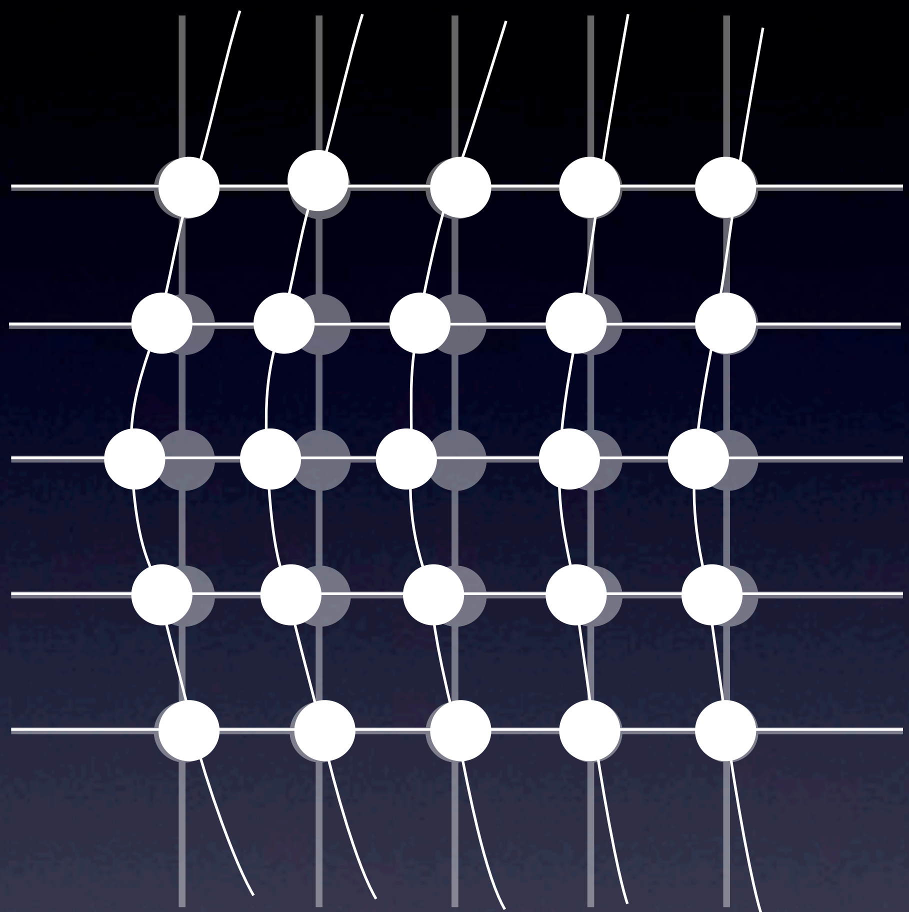


Where are NG mode associated with rotations and boosts?

Nowhere

Spontaneous breaking of non-translationally invariant charges

Low - Manohar's argument

Low, and Manohar ('02)

Ex.: String

order parameter: $\langle \phi(x) \rangle$

trans.: $\langle [P_x, \phi] \rangle = i\partial_x \langle \phi \rangle \neq 0$

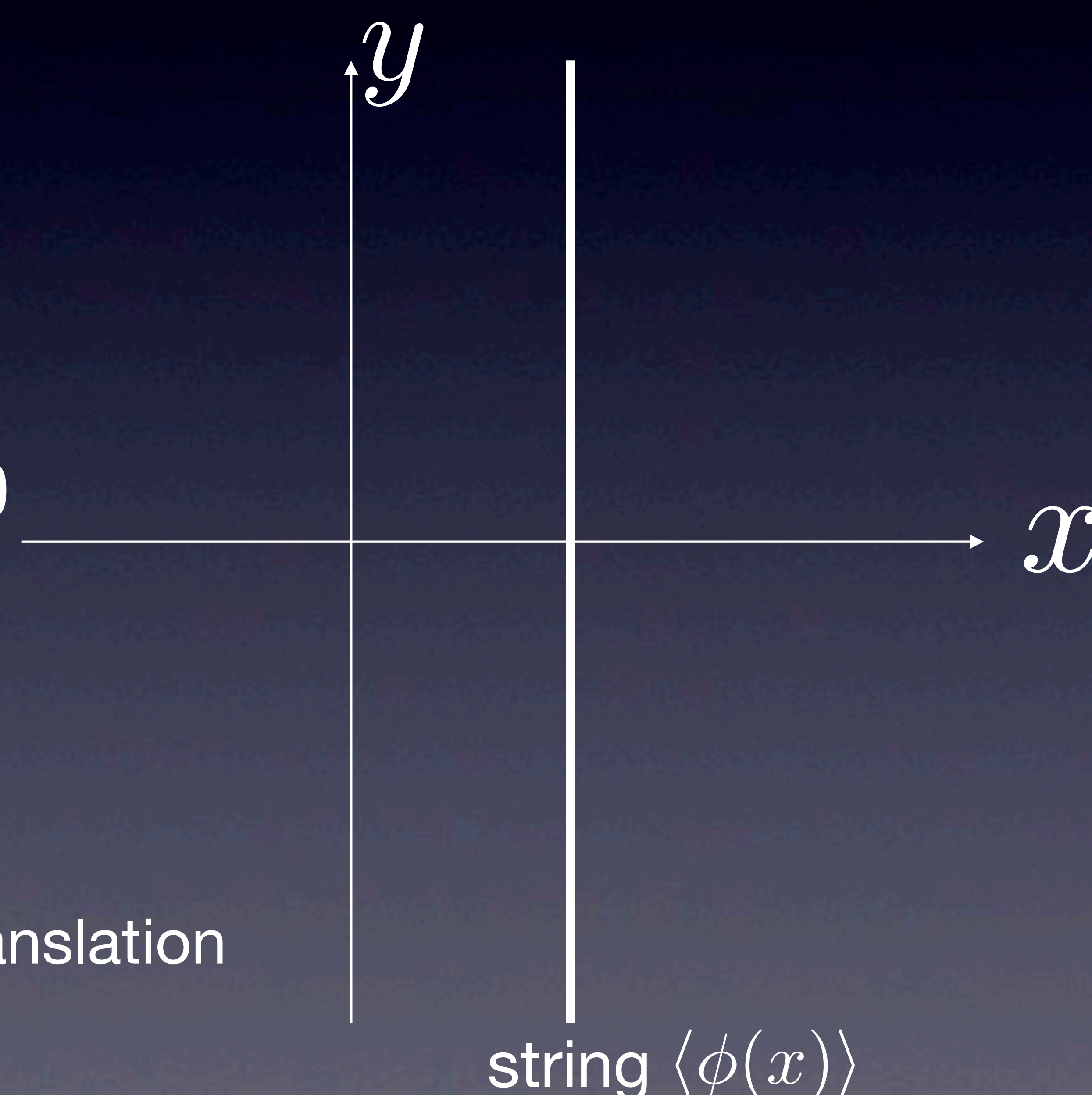
rot.: $\langle [L_z, \phi] \rangle = -iy\partial_x \langle \phi \rangle \neq 0$

Two broken symmetries,

but one NG mode.

Because rotation can be expressed by translation

$$\boldsymbol{L} = \boldsymbol{x} \times \boldsymbol{P}$$



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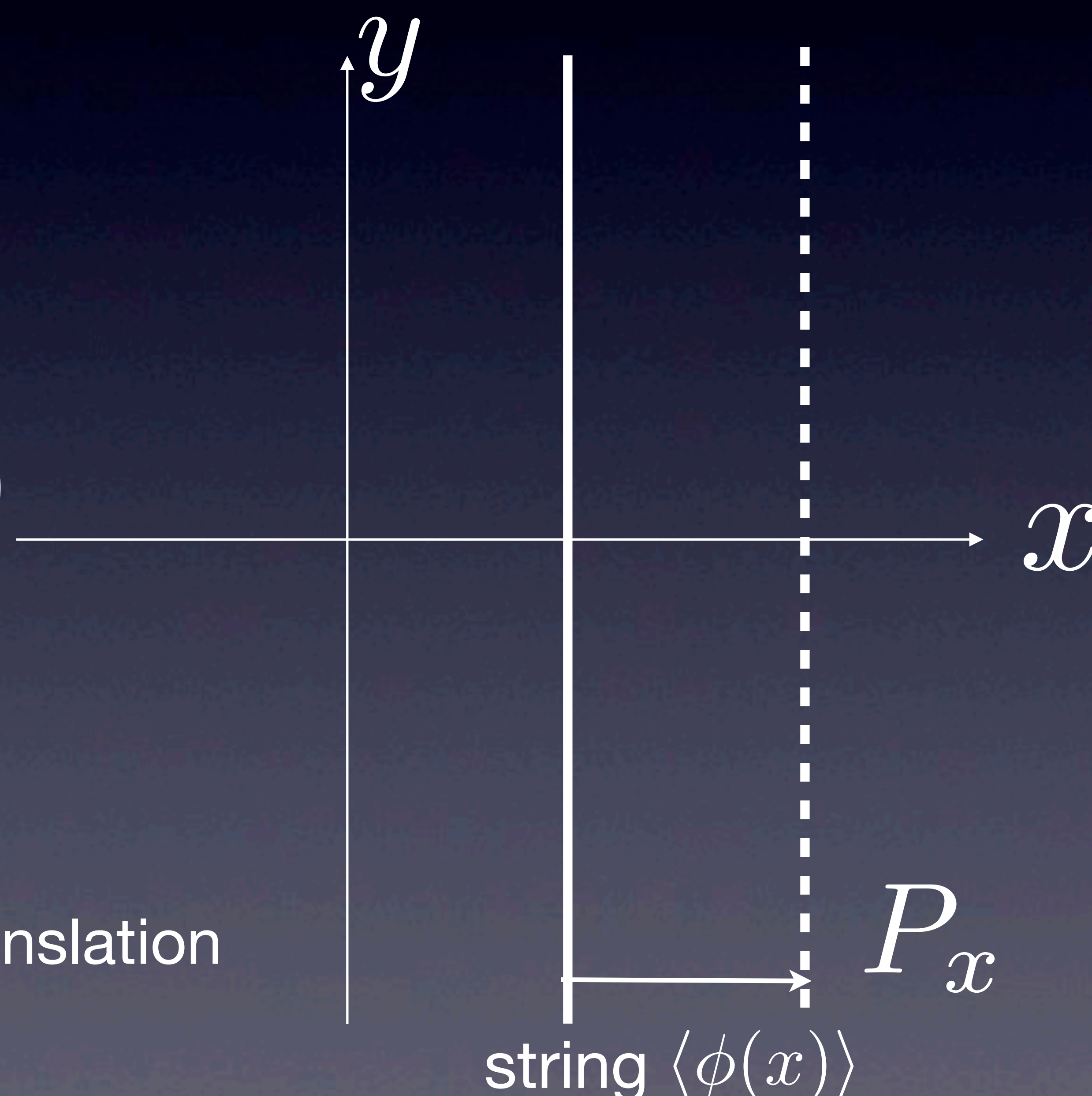

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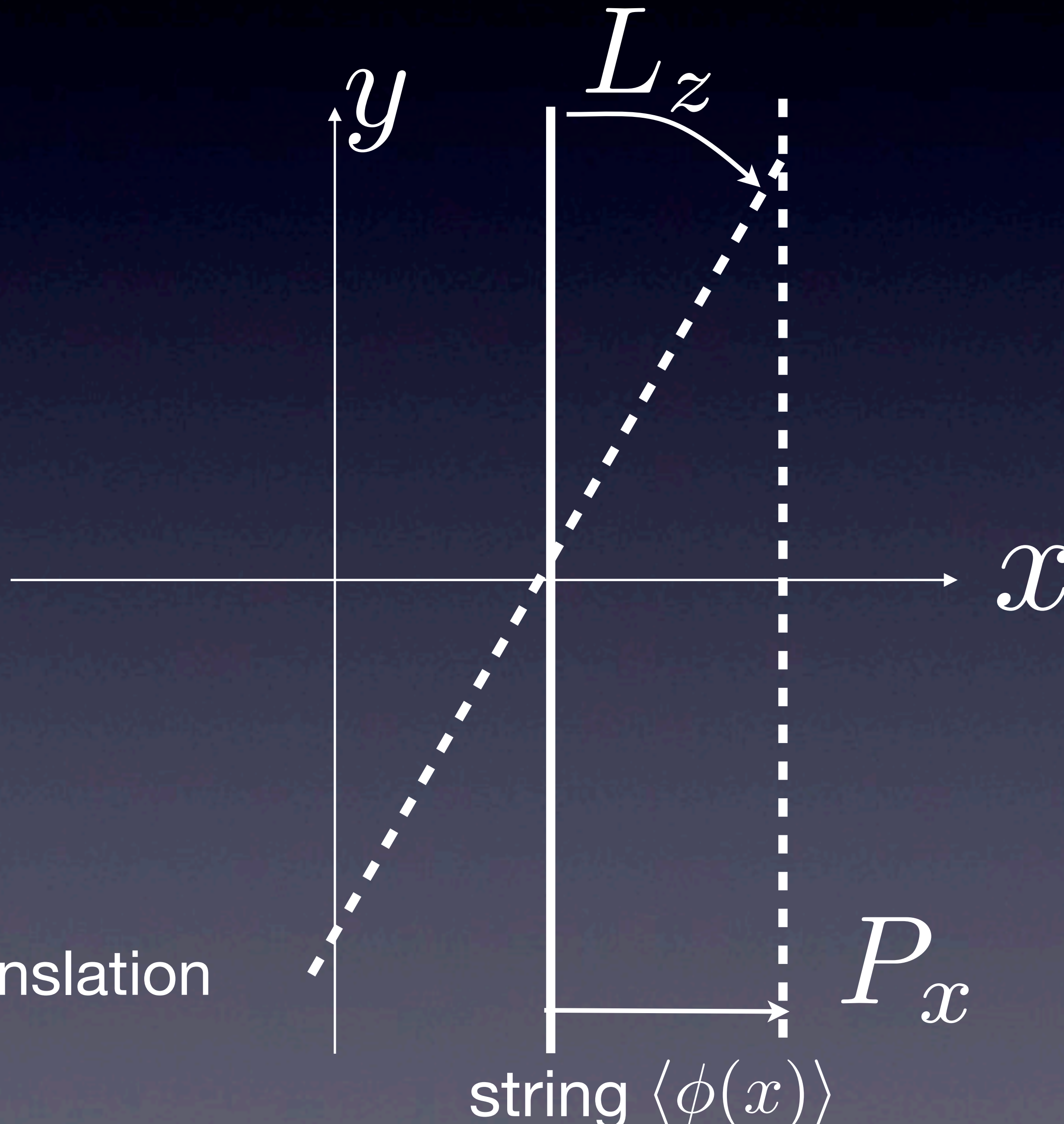

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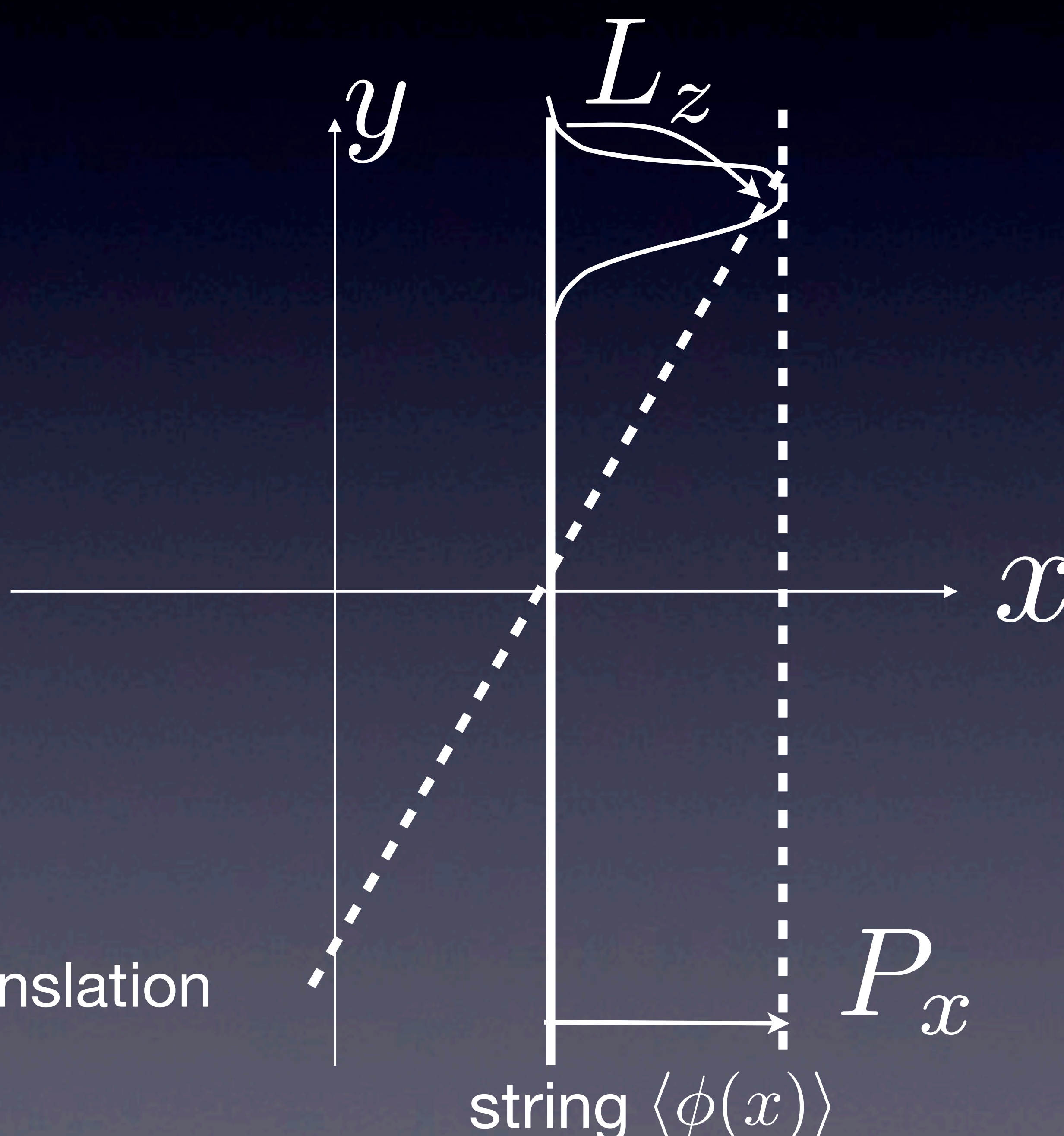


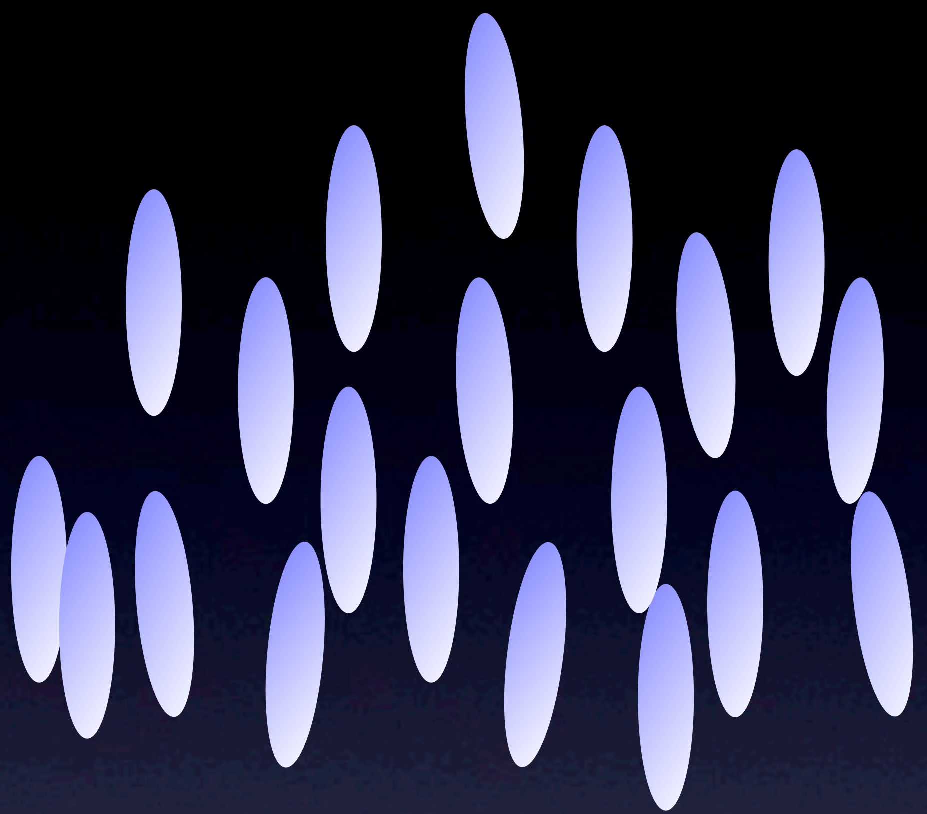
Nontrivial example: Liquid crystal

Nematic phase

Rotation $O(3) \to O(2)$

Two broken generators

Two elastic variables



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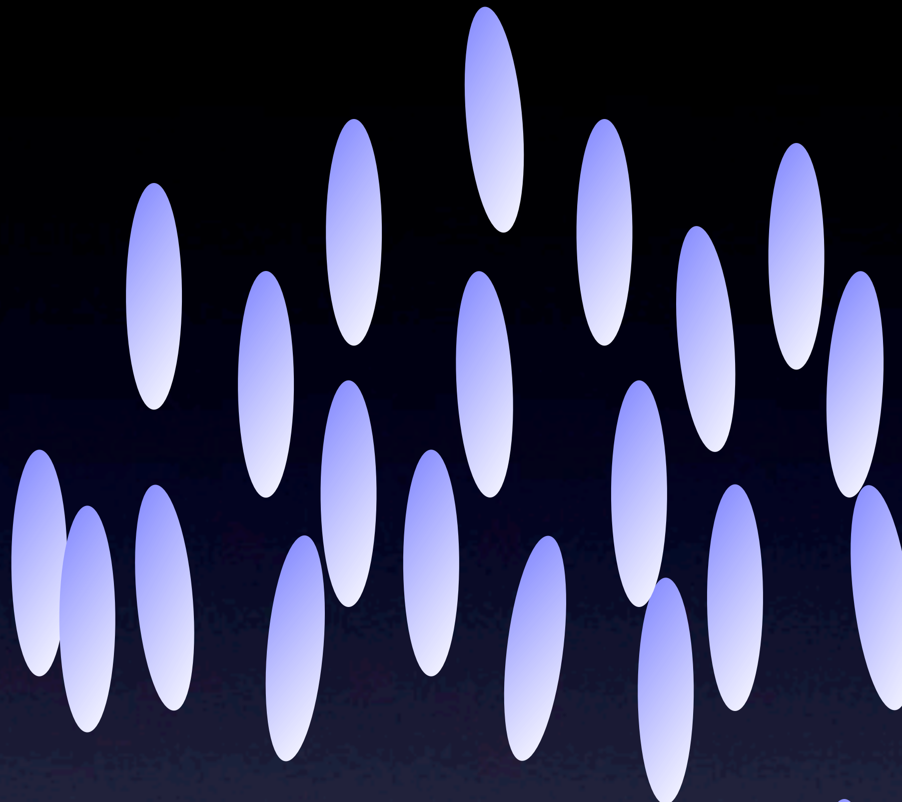
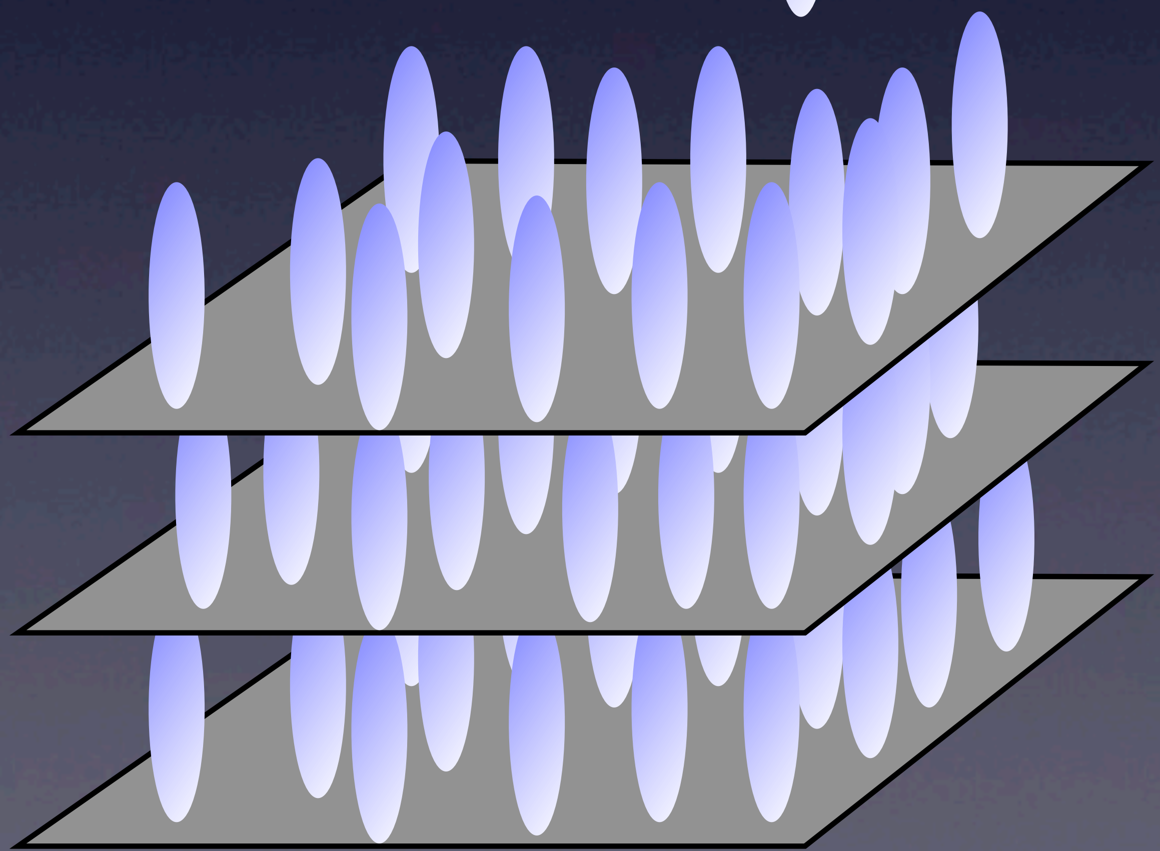


Smectic-A Phase

Rotation $O(3) \to O(2)$
Translation

Three broken generators
One elastic variable



Inverse Higgs mechanism

Ivanov, Ogievetsky ('75), Low, Manohar ('02) Nicolis et al ('13)
Endlich, Nicolis, Penco ('13) Watanabe, Brauner ('14)

$$\xi = e^{ix^\mu P_\mu} e^{iT^a \pi^a(x)}$$

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Maurer-Cartan 1form

$$\alpha = -i\xi^{-1} d\xi = -ie^{-iT^a\pi^a}(d + iP_\mu dx^\mu)e^{iT^a\pi^a}$$

$$= P_\mu dx^\mu + [T^a \pi, iP_\mu dx^\mu + d] + \cdots$$

$$= P_\mu dx^\mu + T^a(\partial_\mu \pi^a + f_\mu{}^{ba}\pi^b)dx^\mu + \cdots$$

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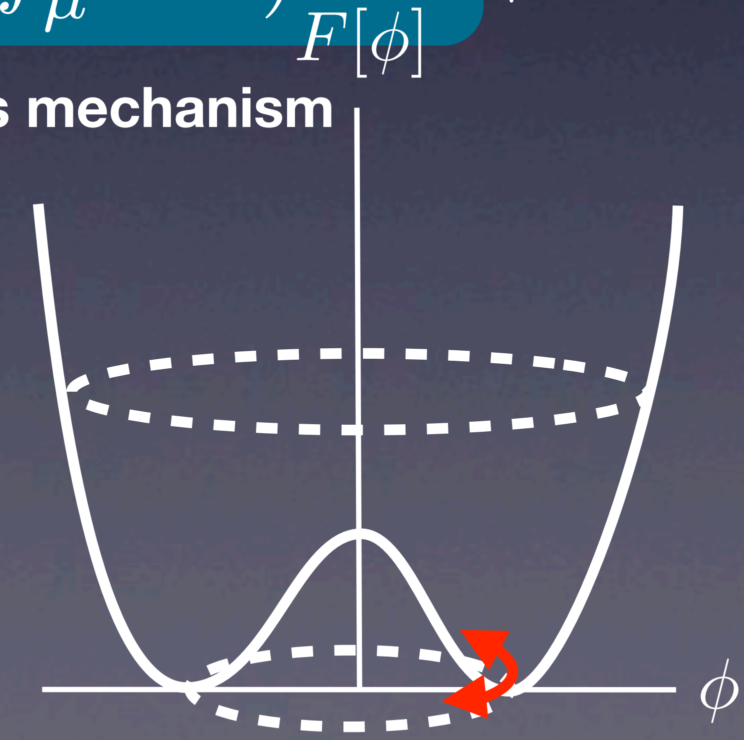

Independence of elastic variables

# of flat direction is not equal to
# of broken symmetry

Hayata, YH ('14)

Ex: QED

Ferrari, Picasso ('71), Hata ('82), Kugo, Terao, Uehara ('85)

Covariant gauge $\mathcal{L}_{\rm GF} = B\partial^\mu A_\mu + \frac{1}{2}\alpha B^2$

Gauge parameter $\theta(x) = a + b_\mu x^\mu$

⇒ charges Q, Q_μ Q_μ is always broken: $\langle [Q_\mu, A_\nu] \rangle = \delta_{\mu\nu}$

Under translation: $[P_\nu, Q_\mu] = i\eta_{\nu\mu} Q$

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$$N_{\rm EV} = N_{\rm NG} = 4$$

NG boson: Photon (2, physical)

scalar and longitudinal parts (unphysical)

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NG boson: Photon (2, physical)

scalar and longitudinal parts (unphysical)

Higgs phase: *Q* is broken.

$\langle [Q, \phi] \rangle = v$ NG higgs (unphysical)

$\langle [Q_\mu, \phi] \rangle = x_\mu v$

➡ $N_{\mathrm{EV}} = N_{\mathrm{NG}} = 1$

Dispersion relation

Ex) Liquid crystal

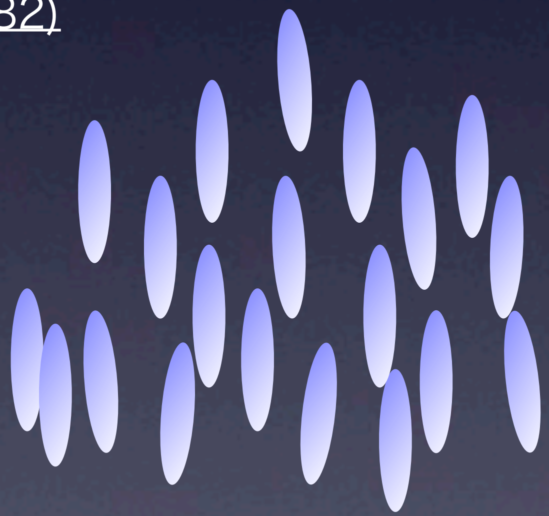
Nematic phase: rotation O(3)→O(2)

$N_{\mathrm{BS}} = N_{\mathrm{EV}} = 2$ $\quad L_i(x) = \epsilon_{ijk} x^j T^{0k}(x) \quad i = 1, 2$

Dispersion relation: $\omega = ak^2 + ibk^2$ Hosino, Nakano('82)

Real and imaginary parts are the same order (damped oscillation)

In case $a = 0$, (overdamping)



Ex) Capillary wave (ripplon)

$$\omega \sim k^{3/2}$$

Summary

For translationally invariant charges

SSB pattern + $\langle [Q_a, Q_b] \rangle$

- Independent elastic variable=N_{BS}
- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle [Q_a, Q_b] \rangle$
- $N_{\text{type-I}} + 2N_{\text{type-II}} = N_{\mathrm{BS}}$
- $N_{\text{type-II}} = \frac{1}{2}\mathrm{rank}\langle [Q_a, Q_b] \rangle$

Type-A (Type-I): $\omega = ak + ibk^2$

Type-B (Type-II): $\omega = ak^2 + ibk^4$

Summary: spacetime breaking

- independent elastic variables ≠ # of broken symmetries (Inverse Higgs mechanism)
- Dispersion relation is not universal (depending on temperature)
- Is there any rule?
- What is the low energy excitation in the quarkyonic matter?